

Of flying frogs and levitrons

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Abstract. Diamagnetic objects are repelled by magnetic fields. If the fields are strong enough, this repulsion can balance gravity, and objects levitated in this way can be held in stable equilibrium, apparently violating Earnshaw's theorem. In fact Earnshaw's theorem does not apply to induced magnetism, and it is possible for the total energy (gravitational + magnetic) to possess a minimum. General stability conditions are derived, and it is shown that stable zones always exist on the axis of a field with rotational symmetry, and include the inflection point of the magnitude of the field. For the field inside a solenoid, the zone is calculated in detail; if the solenoid is long, the zone is centred on the top end, and its vertical extent is about half the radius of the solenoid. The theory explains recent experiments by Geim *et al.*, in which a variety of objects (one of which was a living frog) was levitated in a field of about 16 T. Similar ideas explain the stability of a spinning magnet (LevitronTM) above a magnetized base plate. Stable levitation of paramagnets is impossible.

Samenvatting. Magnetische velden stoten diamagnetische voorwerpen af. Zulke velden kunnen zo sterk zijn dat zij de zwaartekracht opheffen. Het is op deze wijze mogelijk zulke voorwerpen te laten zweven. Dit vormt een stabiel evenwicht, wat in tegenspraak schijnt te zijn met Earnshaw's Theorema. Echter Earnshaw's Theorema is niet langer geldig als het magnetisme veld geïnduceerd is. De totale energie (bevattende bijdragen van het magnetisme en de zwaartekracht) kan toch een lokaal minimum vertonen. Algemene criteria voor zo'n minimum zullen worden opgesteld. Verder zal worden aangetoond dat voor een cilindrisch symmetrisch veld, langs zijn symmetrie as altijd een zone gevonden kan worden waarin een stabiel evenwicht bestaat. Voor het veld binnen een solenoïde zal deze zone in detail bepaald worden. Als deze spoel voldoende lang is bevindt deze zone zich aan het uiteinde van de spoel. De lengte van deze zone langs de symmetrie as van het veld is ongeveer de helft van de straal van de spoel. Deze theorie geeft een goede verklaring voor de experimenten van Geim *et al.* In deze experimenten werden een grote verscheidenheid aan verschillende voorwerpen (waaronder een levende kikker) tot zweven gebracht in velden van ongeveer 16 T. Analoge theorieën verklaren de stabiliteit van een roterend permanent magneetje (LevitronTM) boven een magneetische grondplaat. Het is onmogelijk om paramagnetische voorwerpen stabiel te doen zweven.

1. Introduction

It is fascinating to see objects floating without material support or suspension. In the 1980s, this became a familiar sight when pellets of the new high-temperature type II superconductors were levitated above permanent magnets, and vice versa (Brandt 1989) (levitation of type I superconductors had been achieved much earlier (Arkadiev 1947, Shoenberg 1952)). Recently, two other kinds of magnetic levitation have captured the attention of physicists and the general public. In the LevitronTM (Berry 1996, Simon *et al* 1997, Jones *et al* 1997), a permanent magnet in the form of a spinning top floats above a fixed base that is also permanently magnetized. In diamagnetic levitation, recently achieved by A K Geim with J C Maan, H Carmona and P Main (Rodgers 1997), small objects (live frogs and grasshoppers, waterdrops, flowers, hazelnuts ...) float in the large

(16 T) magnetic field inside a solenoid.

As well as being striking to the eye, magnetic levitation is particularly surprising to physicists because of the obstruction presented by Earnshaw's theorem (Earnshaw 1842, Page and Adams 1958, Scott 1959). This states that no stationary object made of charges, magnets and masses in a fixed configuration can be held in stable equilibrium by any combination of static electric, magnetic or gravitational forces, that is, by any forces derivable from a potential satisfying Laplace's equation. The proof is simple: the stable equilibrium of such an object would require its energy to possess a minimum, which is impossible because the energy must satisfy Laplace's equation, whose solutions have no isolated minima (or maxima), only saddles.

Our purpose here is to explain how stable magnetic levitation of diamagnets can occur despite Earnshaw's theorem. To do this, we obtain formulas for the

energy and equilibrium of a diamagnet in magnetic and gravitational fields (section 2), and then derive the general conditions for the stability of the equilibrium (section 3). Stability is restricted to certain small zones, which we calculate in detail (section 4) for the field inside a solenoid. Finally, we describe (section 5) the diamagnetic levitation experiments carried out by Geim *et al.*

The explanation of the stability of the diamagnets is mathematically related to that of the LevitronTM, but since the LevitronTM has already been treated in several papers we will restrict ourselves here to mentioning the similarities and differences between the two cases. We do not consider the levitation of high-temperature superconductors; this is stabilized by a different mechanism, involving dissipation (dry friction) caused by flux lines jumping between defects that pin them (Brandt 1990, Davis *et al* 1988). Nor do we discuss traps for microscopic particles, some of which are similar to the LevitronTM (Berry 1996) and some of which evade Earnshaw's theorem through time-dependent fields (Paul 1990).

2. Energy and equilibrium

Let the magnetic field inside a vertical solenoid at position $\mathbf{r} = \{x, y, z\}$ be $\mathbf{B}(\mathbf{r})$ (figure 1), with strength $B(\mathbf{r}) = |\mathbf{B}(\mathbf{r})|$, and let the gravitational field have acceleration g . The object that will be levitated in these fields has mass M , volume V (and density $\rho = M/V$), and magnetic susceptibility χ . For diamagnetic materials, $\chi < 0$ (the special case $\chi = -1$ corresponds to superconductors, i.e. perfect diamagnets), so we write $\chi = -|\chi|$. For paramagnets $\chi > 0$, but as we will show in section 3 levitation is impossible for these materials. We will be interested in substances for which $|\chi| \ll 1$. Then, to a close approximation, the induced magnetic moment $\mathbf{m}(\mathbf{r})$ is

$$\mathbf{m}(\mathbf{r}) = -\frac{|\chi|V\mathbf{B}(\mathbf{r})}{\mu_0}. \quad (1)$$

(In a more accurate treatment (Landau *et al* 1984), incorporating the distortion of the ambient field by the object, there is a shape-dependent correction to (1); for a sphere, the r.h.s. is divided by $(1 - |\chi|/3)$. In general, the relation between \mathbf{B} and \mathbf{M} is tensorial.)

By integrating the work $-d\mathbf{m} \cdot \mathbf{B}$ as the field is increased from zero to $\mathbf{B}(\mathbf{r})$, we can obtain the total magnetic energy of the object and, adding this to the gravitational energy, the total energy:

$$E(\mathbf{r}) = mgz + \frac{|\chi|V}{2\mu_0} B^2(\mathbf{r}). \quad (2)$$

For the object to be floating in equilibrium, the total force $\mathbf{F}(\mathbf{r})$ must vanish. Thus

$$\mathbf{F}(\mathbf{r}) = -\nabla E(\mathbf{r}) = -mge_z - \frac{|\chi|V}{\mu_0} B(\mathbf{r})\nabla B(\mathbf{r}) = 0 \quad (3)$$

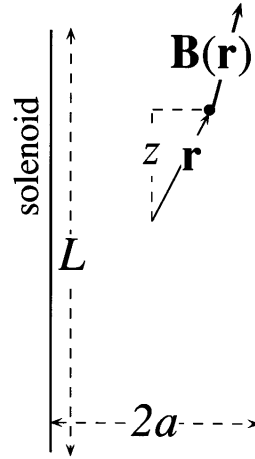


Figure 1. Geometry and notation for field in a solenoid.

where e_z is the upwards unit vector. All the fields we are interested in will have rotation symmetry about e_z (continuous for a solenoid, discrete for the LevitronTM whose base is square). So, considering equilibria on the axis and denoting the field strength by $B(z)$, the equilibrium condition becomes

$$B(z)B'(z) = -\frac{\mu_0\rho g}{|\chi|}. \quad (4)$$

Note that this involves only the density of the levitated object, not its mass.

For the LevitronTM, the spinning-top is magnetized with magnetic moment \mathbf{m} directed along the symmetry axis of the top. The purpose of the spin is to keep \mathbf{m} gyroscopically oriented in the direction for which the force $\nabla\mathbf{m} \cdot \mathbf{B}(\mathbf{r})$ from the base is upwards, that is, with \mathbf{m} antiparallel to the effective dipole representing the base, since unlike dipoles repel (unlike unlike poles). Thus magnetic repulsion can balance gravity. (Without spin, the magnet orients itself parallel to the dipole representing the base, and is therefore attracted to the base, and falls.) The magnetic torque causes \mathbf{m} to precess about the local direction of $\mathbf{B}(\mathbf{r})$. If this precession is fast enough (in comparison with the rate at which the direction of $\mathbf{B}(\mathbf{r})$ changes as the top bobs and weaves during its oscillations about equilibrium), a dynamical adiabatic theorem (Berry 1996) ensures that the angle between \mathbf{m} and $\mathbf{B}(\mathbf{r})$ is preserved. For the LevitronTM, \mathbf{m} is approximately antiparallel to $\mathbf{B}(\mathbf{r})$, so this angle is close to 180° , and the energy is

$$E(\mathbf{r}) = mgz - \mathbf{m} \cdot \mathbf{B}(\mathbf{r}) \approx mgz + |\mathbf{m}|B(\mathbf{r}). \quad (5)$$

Comparing (2) and (5), we see that the energy, and therefore the equilibrium, of both a diamagnetically levitated object and the LevitronTM, depends on the magnitude $B(\mathbf{r})$ of the field; at the end of section 3 we will see that this dependence is crucial to stability in both cases.

3. Stability

For levitation, the equilibrium must be stable, so that the energy must be a minimum, that is, the force $\mathbf{F}(\mathbf{r})$ must be restoring. We begin by showing that this excludes the levitation of paramagnetic objects. A necessary condition for stability is

$$\oiint \mathbf{F}(\mathbf{r}) \cdot d\mathbf{S} < 0 \tag{6}$$

where the integral is over any small closed surface surrounding the equilibrium point. From the divergence theorem, this implies $\nabla \cdot \mathbf{F}(\mathbf{r}) < 0$, and hence, from (2) written for paramagnets, that is with $|\chi|$ replaced by $-\chi$, that

$$\nabla^2 B^2(\mathbf{r}) < 0. \tag{7}$$

But

$$\begin{aligned} \nabla^2 B^2(\mathbf{r}) &= \nabla^2 (B_x^2 + B_y^2 + B_z^2) \\ &= 2 \left[|\nabla B_x|^2 + |\nabla B_y|^2 + |\nabla B_z|^2 \right. \\ &\quad \left. + B_x \nabla^2 B_x + B_y \nabla^2 B_y + B_z \nabla^2 B_z \right] \\ &= 2 \left[|\nabla B_x|^2 + |\nabla B_y|^2 + |\nabla B_z|^2 \right] \geq 0 \end{aligned} \tag{8}$$

where the last equality follows from the fact that the components of \mathbf{B} satisfy Laplace's equation (because there are no magnetic monopoles, so that $\nabla \cdot \mathbf{B} = 0$, and no currents within the solenoid, so that $\nabla \times \mathbf{B} = 0$). Therefore the necessary condition (6) for stability is violated, and stable levitation of paramagnets is impossible. That is why the equations in section 2 were written in the form appropriate for diamagnets.

Equation (8) is the essential step in the proof that the magnitude $B(\mathbf{r})$ of a magnetic field in free space can possess a minimum but not a maximum. This theorem is 'well known to those who know well' (and particularly by physicists who construct traps for microscopic particles) but we do not know who first proved it. It applies to any field that is divergenceless and irrotational. To a good approximation, it applies to velocity fields in the ocean, with the surprising consequence that there is no point within the Pacific Ocean where the water is flowing faster than at all neighbouring points; therefore places where the current has maximum speed lie on the surface.

The sufficient conditions for stability (as opposed to (6), which is merely necessary) are that the energy must increase in all directions from an equilibrium point satisfying (3), that is

$$\partial_x^2 E(\mathbf{r}) > 0 \quad \partial_y^2 E(\mathbf{r}) > 0 \quad \partial_z^2 E(\mathbf{r}) > 0. \tag{9}$$

For diamagnets, it now follows from (2) that

$$\begin{aligned} \partial_z^2 B^2(\mathbf{r}) &> 0 \quad (\text{vertical stability}) \\ \partial_x^2 B^2(\mathbf{r}) &> 0 \quad \partial_y^2 B^2(\mathbf{r}) > 0 \quad (\text{horizontal stability}). \end{aligned} \tag{10}$$

Because of the rotational symmetry, the last two conditions are equivalent. Now we show that the

conditions can be conveniently expressed in terms of the magnetic field on the axis, $B(z)$, and its derivatives $B'(z)$ and $B''(z)$.

We begin by introducing the magnetic potential $\Phi(\mathbf{r})$, satisfying

$$\mathbf{B}(\mathbf{r}) = \nabla \Phi(\mathbf{r}) \tag{11}$$

and its derivatives on the axis

$$\phi_n(z) \equiv \partial_z^n \Phi(0, 0, z). \tag{12}$$

From the fact that Φ satisfies Laplace's equation, and rotational symmetry, there follows

$$\partial_x^2 \Phi(0, 0, z) = \partial_y^2 \Phi(0, 0, z) = -\frac{1}{2} \phi_2(z). \tag{13}$$

Therefore the potential close to the axis can be written

$$\begin{aligned} \Phi(\mathbf{r}) &= \phi_0(z) + \frac{1}{2} (x^2 \partial_x^2 \Phi(0, 0, z) \\ &\quad + y^2 \partial_y^2 \Phi(0, 0, z)) + \dots \\ &= \phi_0(z) - \frac{1}{4} (x^2 + y^2) \phi_2(z) + \dots \end{aligned} \tag{14}$$

From (11), the field strength can now be written

$$\begin{aligned} B^2(\mathbf{r}) &= \phi_1^2(z) + \frac{1}{4} (x^2 + y^2) \\ &\quad \times (\phi_2^2(z) - 2\phi_1(z)\phi_3(z)) + \dots \end{aligned} \tag{15}$$

The stability conditions (10) can now be expressed in terms of $\phi_n(z)$, and thence in terms of the field on the axis:

$$\begin{aligned} D_1(z) &\equiv B'(z)^2 + B(z)B''(z) > 0 \\ &\quad (\text{vertical stability}) \end{aligned} \tag{16}$$

$$\begin{aligned} D_2(z) &\equiv B'(z)^2 - 2B(z)B''(z) > 0 \\ &\quad (\text{horizontal stability}). \end{aligned}$$

For the LevitronTM, where the magnetic energy (5) depends on $B(\mathbf{r})$ rather than $B^2(\mathbf{r})$, a similar analysis leads to the same horizontal stability condition, and the simpler vertical stability condition $B''(\mathbf{r}) > 0$.

Mathematically, the reason why diamagnets and the LevitronTM can be levitated in spite of Earnshaw's theorem is that the energy depends on the field strength $B(\mathbf{r})$, which unlike any of its components does not satisfy Laplace's equation and so can possess a minimum. Physically, the diamagnet violates the conditions of the theorem because its magnetization \mathbf{m} is not fixed but depends on the field it is in, via (1). Microscopically, this is because diamagnetism originates in the orbital motion of electrons and so is dynamical. In the LevitronTM, the magnitude of \mathbf{m} is fixed but its direction is slaved to the direction of $\mathbf{B}(\mathbf{r})$ by an adiabatic mechanism that is also dynamical (at the macroscopic level) because it relies on the fast precession of the top.

The (non-dissipative) stability of permanent magnets levitated above a (concave upwards) bowl-shaped base of type I superconductor (e.g. lead) (Arkadiev 1947) is similar to that of the diamagnets we have been considering. The superconductor is a perfect diamagnet ($\chi = -1$), and so the permanent magnet above it is repelled by the field of the image it induces (Saslow 1991). If the magnet moves sideways, the image gets closer, so that the energy increases.

4. Stable zones

On the axis of a solenoid, or above the base of a Levitron™, the field $B(z)$ decreases monotonically as z increases from 0 to ∞ , and there is an inflection point at some height z_i , that is $B''(z_i) = 0$. At z_i , both discriminants D_1 and D_2 in (16) are obviously positive, so the equilibrium is stable at z_i . Simple geometrical arguments show that D_1 has a zero at a point $z_1 < z_i$, and vertical stability requires $z > z_1$; similarly, D_2 has a zero at a point $z_2 > z_i$, and horizontal stability requires $z < z_2$. This establishes the existence of a stable zone on the axis, namely $z_1 < z < z_2$, within which diamagnetic objects can be levitated.

It is necessary for the equilibrium position satisfying (4) to lie in the stable zone. This can be achieved by changing the current in the solenoid, which scales the magnetic field strength $B(r)$ while preserving the geometry of the field lines and therefore the stable zone determined by (16).

In the Levitron™, the stable zone is $z_i < z < z_2$, and, since the base is a permanent magnet whose field cannot easily be altered, the equilibrium height of the floating top can be brought into this interval by adding or removing small washers to change the weight Mg .

As a model to study in detail, we consider the field inside a long solenoid of length L and radius a (figure 1). Then, defining the scaled variables

$$\begin{aligned} \xi &\equiv x/a, & \eta &\equiv y/a, \\ \zeta &\equiv z/L & \text{and } \delta &\equiv 2a/L \end{aligned} \quad (17)$$

and the field B_0 at the centre of the solenoid $z = 0$, we have, introducing obvious notations,

$$\begin{aligned} \frac{B(\zeta, \delta)}{B_0} &\equiv \mathbf{B}(\zeta, \delta) = \frac{1}{2}\sqrt{1 + \delta^2} \\ &\times \left(\frac{1 - 2\zeta}{\sqrt{(1 - 2\zeta)^2 + \delta^2}} + \frac{1 + 2\zeta}{\sqrt{(1 + 2\zeta)^2 + \delta^2}} \right). \end{aligned} \quad (18)$$

There are inflections close to the ends $\zeta = \pm 1/2$ of the solenoid; levitation occurs near the top end, that is $\zeta = +1/2$, where the field gradient is negative as required by (4). Figure 2 illustrates this field, and the corresponding discriminants (16), for $\delta = 0.1$. The stable zone is $\zeta_1 = 0.487083 < \zeta < \zeta_2 = 0.510223$.

For thin solenoids ($\delta \ll 1$), some simplification is possible, since then the second term in (18) can be approximated by unity near $\zeta = 1/2$. A short analysis shows that in this limit the inflection and stable zone are, when expressed in the original z coordinate,

$$\left. \begin{aligned} z_i &= \frac{1}{2}L \\ z_1 &= \frac{1}{2}L - 0.258199a < z < z_2 = \frac{1}{2}L + 0.204124a \end{aligned} \right\} \quad (L \gg a). \quad (19)$$

For fat solenoids ($\delta \gg 1$), simplification is again possible, because then the field is that on the axis of a current loop, namely

$$B(z) = \frac{B_0}{(1 + (z/a)^2)^{3/2}} \quad (a \gg L). \quad (20)$$

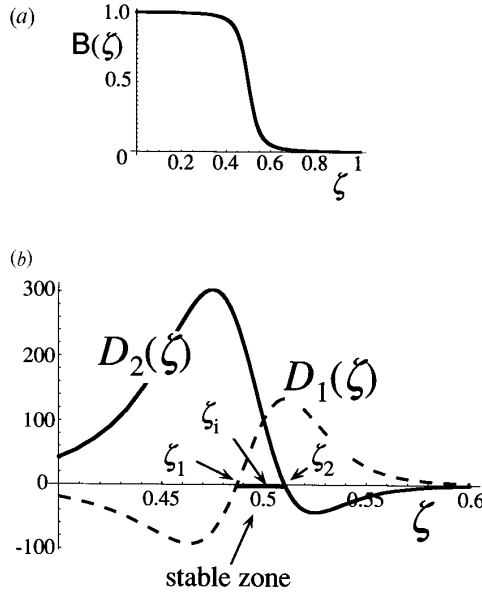


Figure 2. (a) Field on the axis inside a solenoid with $\delta = 2a/L = 0.1$; (b) the discriminants $D_1(\zeta)$ and $D_2(\zeta)$ defined by (16), and the stable zone where both are positive.

From (16), the inflection and stable zone are

$$\left. \begin{aligned} z_i &= \frac{1}{2}a \\ z_1 &= \frac{1}{\sqrt{5}}a = 0.378a < z < z_2 = \sqrt{\frac{2}{5}}a = 0.6325a \end{aligned} \right\} \quad (a \gg L). \quad (21)$$

By Ampère’s equivalence between distributions of magnetization and current loops, the field (20) is the same as that on the axis of a uniformly magnetized disc. Therefore, with the vertical stability condition $B''(r) > 0$ (see the remark following equation (16)), (21) leads to the stable zone previously calculated (Berry 1996) for a Levitron™ with a circular disc base, namely $a/2 < z < a\sqrt{(2/5)}$. (If the base of the Levitron™ is a ring, rather than a disc, the stable region is much higher, namely $1.6939a < z < 1.8253a$, and this explains the operation of the recently developed ‘superlevitron’.)

It is instructive to display spatial contour maps of the energy (2) as the field B_0 at the centre of the solenoid is varied, showing the appearance and disappearance of the minimum as the equilibrium enters and leaves the stable zone. We employ the dimensionless field β and energy E defined by

$$\begin{aligned} B_0^2 &\equiv \beta^2 \frac{\rho g L \mu_0}{|\chi|} \\ E(r) &\equiv \frac{|\chi| V B_0^2}{2\mu_0} E(\xi, \eta, \zeta; \beta, \delta) \end{aligned} \quad (22)$$

where, in terms of (15) and the field profile (18),

$$E(\xi, \eta, \zeta; \beta, \delta) \equiv \frac{2}{\beta^2} \zeta + \frac{1}{4} [\mathbf{B}(\zeta, \delta)^2 + \frac{1}{4} (\xi^2 + \eta^2)] \times \{\mathbf{B}'(\zeta, \delta)^2 - 2\mathbf{B}(\zeta, \delta)\mathbf{B}''(\zeta, \delta)\} \quad (23)$$

(the primes denote $\partial/\partial\zeta$). From the equilibrium condition (4), the field $\beta(z)$ for which the diamagnet floats at height ζ is

$$\beta(z)^2 = -[\mathbf{B}(\zeta, \delta)\mathbf{B}'(\zeta, \delta)]^{-1}. \quad (24)$$

Figure 3 shows the E landscape as the field β is decreased through the stable range, for a solenoid with $\delta = 0.1$. At the top of the range (figure 3(b)) $\beta = \beta_2 = 0.513563$, corresponding to equilibrium at the upper limit $z = z_2$ of the stable zone, and at the bottom of the range (figure 3(d)) $\beta = \beta_1 = 0.417998$, corresponding to equilibrium at the lower limit $z = z_1$ of the stable zone. At β_2 the minimum is born (along with two off-axis saddles) from the splitting of an axial saddle; at β_1 , the minimum dies as it annihilates with another axial saddle. We caution against quantitative reliance on the details of these landscapes near the wall of the solenoid (e.g. near $\xi = 0.05$ in figure 3), because they are based on the quadratic approximation (23), which is strictly valid only close to the axis.

Stably levitated diamagnets can make small, approximately harmonic, oscillations near the energy minimum, and these are observed as the gentle bobbing and weaving of the objects. Larger oscillations will be anharmonic. The region they explore has the form of a conical pocket (figure 3(c)), in which motion is almost certainly nonintegrable and probably chaotic. We think this would repay further study, but here confine ourselves to estimating the greatest lateral extent of the region in which the oscillations occur. From figure 3, it is reasonable to define this as the distance $R = \sqrt{(x^2 + y^2)}$ from the axis to the off-axis saddles for the field that corresponds to equilibrium at z_i , namely $\beta = 0.445301$. It follows from (23) that these saddles lie at $z = z_2$, and use of (4) then leads to

$$R^2 = 4L^2 \frac{[\mathbf{B}(\zeta_i, \delta)\mathbf{B}'(\zeta_i, \delta) - \mathbf{B}(\zeta_2, \delta)\mathbf{B}'(\zeta_2, \delta)]}{\mathbf{B}(\zeta_2, \delta)\mathbf{B}''(\zeta_2, \delta)}. \quad (25)$$

For thin solenoids, this can be evaluated as

$$R = 0.75569a \quad (L \gg a). \quad (26)$$

When $\delta = 0.1$ this gives $R/L = 0.0377$, in agreement with figure 3(c) (which was calculated without the thin-solenoid approximation).

5. Experiment

Most diamagnetic materials have susceptibilities of order $\chi \approx -10^{-5}$. For water, $\chi = -8.8 \times 10^{-6}$ (Kaye and Laby 1973), and using $\rho = 1000 \text{ kg m}^{-3}$

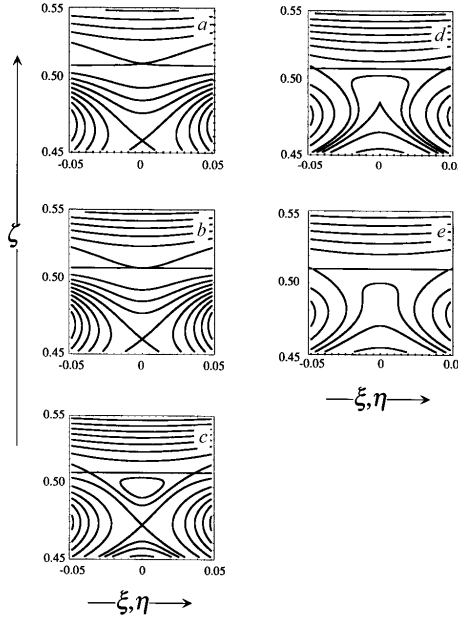


Figure 3. Contours of the scaled energy $E(\xi, \eta, \zeta; \beta, \delta)$ (gravitational + magnetic) for a diamagnet, for different values of the dimensionless field β (defined by (22)) at the centre of a solenoid with $\delta = 0.1$.

- (a) $\beta = 0.527046$;
- (b) $\beta = \beta_2 = 0.513563$, i.e. levitation at z_2 ;
- (c) $\beta = 0.445301$, i.e. levitation at z_i ;
- (d) $\beta = \beta_1 = 0.417998$;
- (e) $\beta = 0.411693$, i.e. levitation at z_1 .

the equilibrium condition (4) gives the required product of field and field gradient as

$$B(z)B'(z) = -1400.9 \text{ T}^2 \text{ m}^{-1}. \quad (27)$$

This has been achieved in experiments involving one of us (Geim *et al*) with a Bitter magnet whose geometry is shown in figure 4(a). The operation of this electromagnet consumed 4 MW, but we emphasize that this is power dissipated in the coils, not power required for levitation—indeed, with the field of a persistent current in a superconducting magnet levitation can be maintained without supplying any energy.

The measured field profile is shown in figure 5. The inflection point is at $z_i = 78 \text{ mm}$, where the field is $B(z_i) = 0.63B_0$ and the gradient of the field at z_i is $-8.15B_0 \text{ T m}^{-1}$, from which the required central field is predicted via (27) to be

$$B_0 = 16.5 \text{ T}. \quad (28)$$

From the measured data we have calculated the discriminants D_1 and D_2 defined by (16), and thence the stable zone, which is predicted to be $z_1 = 67.5 \text{ mm} < z < z_2 = 87.5 \text{ mm}$.

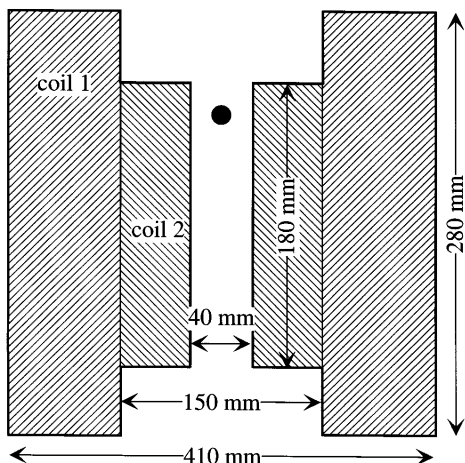


Figure 4(a). Geometry of coils in Bitter magnet used for levitating diamagnetic objects. The currents in the two coils were equal. The region of stable levitation is near the top of coil 1, and marked with a dot.

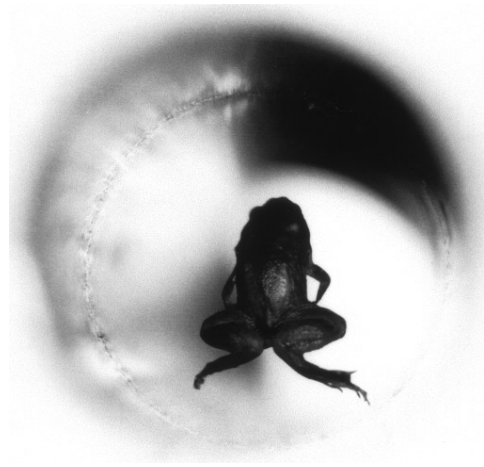


Figure 4(b). Frog levitated in the stable region.

A variety of diamagnetic objects was inserted into the magnet, and the current through the coils adjusted until stable levitation occurred (figure 4(b)). The corresponding fields B_0 were all close to the calculated 16 T, and the objects always floated near the top of the inner coil, as predicted. Careful observation of a (3 mm diameter) plastic sphere showed that it could be held stably in the range $(69 \pm 1) \text{ mm} < z < (86 \pm 1) \text{ mm}$, in very good agreement with theory.

The induced dipole m (equation (1)) responsible for the levitation of a diamagnet can be regarded as

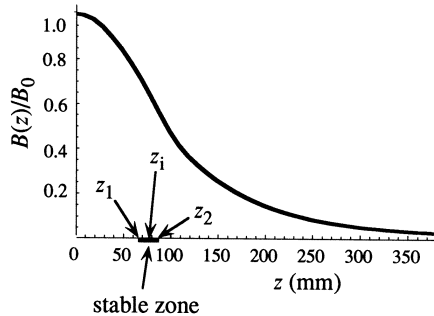


Figure 5. Profile of field on axis of Bitter magnet in figure 4, measured at intervals of 10 mm, showing the stable zone near the top of coil 1.

equivalent to a current $I = |m|/A$ circulating in a loop of area A embracing it. For an object of radius 10 mm, such as the very young frog that was levitated (figure 4(b)), this current is about 1.5 A (corresponding to a field $B \approx 10^{-5} B_0 \approx 1.5$ Gauss induced inside the frog). Of course this represents the summation of microscopic currents localized in atoms, not the bulk transport of charge, so the living creatures were not electrocuted. Indeed, they emerged from their ordeal in the solenoid without suffering any noticeable biological effects—see also Schenck (1992) and Kanal (1996).

As we showed earlier, it is impossible to levitate paramagnets stably. Balance of forces can however be achieved, and from (4) with the sign reversed it is clear that this occurs for $z < 0$, and close to the centre of the solenoid—rather than near the bottom—because $\chi_{\text{paramagnetic}} \approx 10^{-3} \approx 100 \chi_{\text{diamagnetic}}$; this position is vertically stable but laterally unstable. Nevertheless, some paramagnetic objects (Al, several types of brass, stainless steel, paramagnetic salts with Mn and Cu) were suspended in this way, but not levitated: they were held against the side wall of the inner coil. On a few occasions, paramagnets floated without apparent contact, but were found to be buoyed up by a rising current of paramagnetic air; when this was inhibited, for example by covering the ends of the solenoid with gauze, the objects slipped sideways and were again held against the wall.

6. Discussion

Our treatment of diamagnetic levitation has neglected at least three small effects that could have interesting consequences. The first arises from the shape-dependence of the induced magnetic moment. For living organisms (e.g. frogs) trapped in the energy minimum this could be exploited to provide an escape mechanism. If the frog is initially in equilibrium, there are no forces

on it. By changing shape (e.g. from a sphere to an ellipsoid) the induced moment will change (Landau *et al* 1984), and the force will no longer be zero, so the frog will start to oscillate about a slightly different point. By repeating this manoeuvre at the frequency of oscillations in the minimum, the oscillations will be amplified by parametric resonance until the frog leaves the stable zone. This is a tiny effect, because the shape-dependence of m is of the order $|\chi| \approx 10^{-5}$, so escape would require 10^5 such 'swimming strokes'; therefore the frog would have to be persistent as well as highly coordinated. (In practice, the frog does try to swim—but in the ordinary way, by paddling the air in the solenoid—but nevertheless remains held in the energy minimum, for the entire observation—up to 30 minutes.)

The second effect arises from the finite extent of any real levitated object. Its equilibrium depends on the total magnetic force, which must balance the weight. The local force balance (4) will occur only at one height z_b in the body. For $z < z_b$, the net force on each element will be upwards, and for $z > z_b$ the net force will be downwards. Therefore the object will be compressed to an extent that depends on how much BB' varies across it, that is on the curvature of $B^2(z)$ at z_b . A land-based living creature would be unlikely to feel this effect, since it is already accustomed to a much greater inhomogeneity: the external upward force that balances gravity is concentrated in a molecular layer in the soles of its feet.

The third effect occurs for objects that are diamagnetically inhomogeneous, so that their different parts (e.g. flesh and bone for a living organism) have different χ s. Then, as just described for an extended object, the force balance will be different at different points. This could cause strange sensations; for example, if $|\chi|_{\text{flesh}} > |\chi|_{\text{bone}}$ the creature would be suspended by its flesh with its bones hanging down inside, in a bizarre reversal of the usual situation that could inspire a new (and expensive) type of face-lift (since $|\chi|_{\text{bone}} \approx |\chi|_{\text{water}}$ (Schenck 1992) this would require $|\chi|_{\text{flesh}} > |\chi|_{\text{water}}$).

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