Black hole heat engine

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Two black holes can merge to create a bigger black hole, thus increasing the entropy of the universe. Alternatively, they can be used as two heat reservoirs from which work can be extracted. We discuss a process during which two black holes are transformed into one while the total entropy is kept as constant. The resulting black hole has a smaller mass than the total mass of the input black holes and the mass difference is converted into work. Although the process will probably not be used within the next 10^{11} yr for energy production, we can speculate that it might be an energy source for those who might inhabit our universe after that. We discuss the basic thermodynamics of the proposed system. © *2012 American Association of Physics Teachers.*
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I. INTRODUCTION

Renewable energy resources will not be renewable forever. Even the Sun as well as all the stars will ultimately burn all their nucleosynthetic fuel. Will life in any form be possible after that? Instead of global warming, future inhabitants of our expanding universe will face universal cooling. In an almost empty universe with just background radiation of decreasing temperature and with sparsely scattered remnants of dead stars and galaxies, what could serve as an energy source to continue anything similar to life? And where can the waste entropy be dumped to keep organized structures possible? Questions such as these have been put forward by Frautschi[1] and by Krauss and Starkman.[2] Even though the prognosis is not very optimistic for times beyond $\approx 10^{50}$–10^{100} yr, the good news is that before all the black holes evaporate by Hawking radiation, an interesting option is found by merging black holes. The time for exploiting this resource will exceed by many orders of magnitude the present age of our universe.

As suggested by Bekenstein,[3] a black hole has entropy which is proportional to the square of its mass,

$$S_{\text{BH}} = 4\pi k \left(\frac{M}{m_P}\right)^2, \tag{1}$$

where $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, M is the mass of the black hole, $m_P = \sqrt{\hbar c/G} \approx 2.2 \times 10^{-8}$ kg is the Planck mass, G is the gravitation constant, c is the speed of light, and $\hbar$ is the reduced Planck constant. When a star collapses to form a black hole, a huge amount of entropy is produced. Because the entropy is proportional to M^2, merging two black holes of masses M_1 and M_2 into one of mass $M_1 + M_2$ leads to a total entropy increase proportional to $2M_1M_2$. To put the entropy increase in perspective, we note that for a human to keep his or her metabolism for a lifetime increases the entropy of the universe by about 10^9 J/K, mostly by converting the chemical energy of food into thermal energy.[4] The Earth during its existence has contributed to the entropy production by about 10^{32} J/K, mostly by converting each incoming solar photon into ~ 17 infrared photons radiated into space.[5] The Sun has contributed an entropy increase of about 10^{40} J/K, which came into being by producing about 5×10^6 photons emitted into space per nucleon subject to nucleosynthesis.[6] If the solar mass is squeezed to become a black hole, the entropy would increase to 10^{54} J/K, that is, by fourteen orders of magnitude. Even if the Sun consumed all its nuclear fuel in the most efficient way and converted all its hydrogen into iron, dispensing its energy by radiating visible light, the entropy produced would be just $\sim 3 \times 10^{41}$ J/K, which is negligible compared with the entropy increase of the black hole formation. If two solar-mass black holes merge, the total entropy increases by twice the entropy of a single solar-mass black hole $\sim 10^{54}$ J/K.

Disposing of entropy is essential for living organisms.[7] If the organism has to keep its entropy low, then the entropy must increase somewhere else in the universe. Feeding black holes seems to be a very efficient way to get rid of excess entropy. References 1 and 2 deal with the question of how the possibilities for dumping excess entropy produced by maintaining life in whatever form could continue in the aging universe. Although the entropy bookkeeping is clear, we are left to speculate by what mechanism the suggested "black hole amalgamation"[1] could be used for anything useful.

In this paper, we discuss a method to produce work by running a heat engine between two black holes that serve as thermal reservoirs. Working with Carnot efficiency, the two black holes can ultimately merge into one, keeping the total entropy unchanged. The resulting black hole thus must have a smaller mass than the total mass of the input black holes. The difference corresponds to the extracted work. As we will show, by merging two black holes with masses M_1 and M_2, we can in principle extract up to $(M_1 + M_2 - \sqrt{M_1^2 + M_2^2})c^2$ of useful work.

Black hole thermodynamics has been discussed extensively since the seminal papers by Bekenstein[3] and Hawking.[8,9] However, not much attention has been paid to heat engine methods. A model suggested by Kaburaki and Okamoto[10] uses a Kerr black hole as the working medium in a Carnot-like engine running between two reservoirs formed by boxes with radiation. Recently Deng and Gao proposed a Carnot engine with radiation as the working fluid and a black hole as the cold reservoir.[11]

In this paper, we discuss a simple model suitable for undergraduate courses. It combines the basic ideas of thermodynamics and black hole physics, can serve as an excursion into times far exceeding present cosmological scales, and can give students a flavor of the energy problem viewed as "sub specie aeternitatis," or close to that.

II. HEAT RESERVOIR: BLACK HOLE AND RADIATION IN A BOX

We consider a simple system consisting of a Schwarzschild black hole (nonrotating and uncharged) inside a box filled with radiation. Let the inside walls of the box be perfectly reflecting, thus isolating the interior from the outside world so that the total energy in the box is constant. In this way, the black hole is in thermodynamic equilibrium with the radiation.

The mechanism of black hole radiation has been described by Hawking[8,9] and the composition of the radiation has been discussed by Page.[12] For simplicity, we assume a model where the radiation is entirely due to photons. The real composition of the radiation would influence the numerical values of the parameters, but not the scaling of the relevant quantities.

The total energy of the system is

$$E_{\text{tot}} = Mc^2 + aVT^4, \tag{2}$$

where the first term is the black hole energy and second term is the energy of the radiation with

$$a = \frac{\pi^2 k^4}{15 c^3 \hbar^3}, \tag{3}$$

V is the volume of the box and T is the radiation temperature. The black hole temperature is (see, for example, Ref. 13)

$$T_{\text{BH}} = \frac{\partial E_{\text{BH}}}{\partial S_{\text{BH}}} = \frac{\hbar c^3}{8\pi k G M}. \tag{4}$$

Note that the Schwarzschild radius R_S of a black hole of temperature T_{BH} is similar in size to the typical wavelength of thermal radiation at the same temperature,

$$R_S = \frac{2GM}{c^2} = \frac{\hbar c}{4\pi k T_{\text{BH}}}. \tag{5}$$

The entropy of the combined system is

$$S_{\text{tot}} = S_{\text{BH}} + S_{\text{rad}}, \tag{6}$$

with the radiation entropy

$$S_{\text{rad}} = \frac{4}{3} aVT^3. \tag{7}$$

By using Eq. (2), we can express the radiation temperature in terms of the total energy and the black hole mass such that the total entropy is a function of M

$$S_{\text{tot}} = k\left[\frac{4\pi G}{\hbar c}M^2 + \frac{4}{3}\sqrt[4]{\frac{\pi^2 V c^3}{15\hbar^3}}(M_{\text{tot}} - M)^{3/4}\right], \tag{8}$$

with $M_{\text{tot}} \equiv E_{\text{tot}}/c^2$ (see Fig. 1). The equilibrium condition corresponds to the extrema of the total entropy. A local maximum is at the boundary $M = 0$ (no black hole, all the energy is in radiation). We set the partial derivative equal to zero,

$$\left(\frac{\partial S_{\text{tot}}}{\partial M}\right)_{V, E_{\text{tot}}} = 0, \tag{9}$$
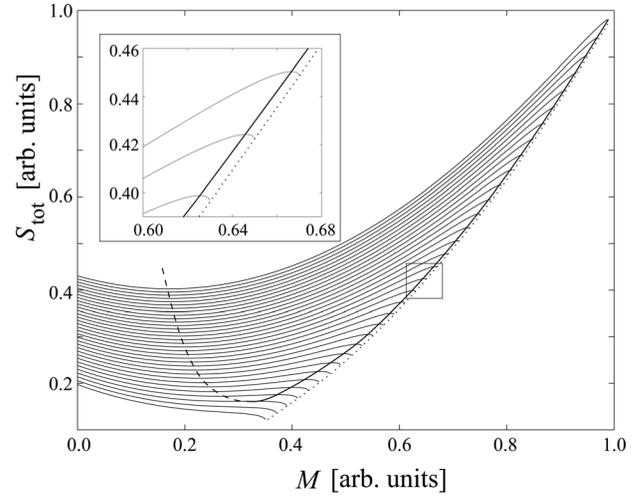
and find the condition for other local extrema,

Fig. 1. Dependence of the entropy S_{tot} of a system composed of a black hole and radiation inside a box with insulating walls on the mass of the black hole M, Eq. (8). The various lines correspond to varying total energies while the volume of the box is fixed. The dotted line represents the boundary $M = M_{\text{tot}}$ and the dashed line joins points corresponding to unstable equilibrium. The thick full line joins points of stable (or metastable, if states of pure radiation with $M = 0$ have more entropy) equilibrium with the black hole dominating the system.

$$M^4(M_{\text{tot}} - M) = \frac{\pi^2 c^7 \hbar V}{15(8\pi G)^4}. \tag{10}$$

Equation (10) has real solutions between 0 and M_{tot} only for sufficiently small V,

$$V \leq V^* = \frac{2^{20} 3\pi^2 G^4 M_{\text{tot}}^5}{5^4 c^7 \hbar}. \tag{11}$$

For larger volumes the black hole is unstable and eventually evaporates into radiation. For volumes $V < V^*$, there are two local extrema corresponding to the black hole temperature equaling the radiation temperature. The equilibrium for the smaller value of M is unstable (S_{tot} has a local minimum). In this case the black hole would either completely evaporate resulting in the radiation-only case, or would grow and absorb radiation and end up in the other, stable equilibrium with the larger value of M.

The condition (11) defines the length scale of the box in which the black hole can be kept in thermal equilibrium,

$$\ell_{\text{box}} \lesssim 40 \ell_P \left(\frac{M}{m_p}\right)^{5/3} \approx 20 R_S \left(\frac{M}{m_p}\right)^{2/3}, \tag{12}$$

where $\ell_P = \sqrt{G\hbar/c^3} \approx 1.6 \times 10^{-35}$ m is the Planck length.

The dynamics of black holes approaching thermal equilibrium in a box with radiation has been studied by Custodio and Horvath.[14]

In the following, we choose the system in stable equilibrium to be the thermal reservoir for the Carnot process. For simplicity, we will assume that the box is always so small that most of the energy is in the black hole. For a sufficiently small volume the fraction of energy in the radiation is $\sim (4^4/5^5)(V/V^*) \approx 0.08 V/V^*$. Equation (12) implies that for black holes with a mass much larger than the Planck mass the box can also be made sufficiently large so that on its surface the gravitational red shift [$\approx R_S/(2\ell)$] is negligible

(ℓ is the distance between the black hole center and a point on the box surface).

III. CARNOT PROCESS

The method is shown in Fig. 2. There are two boxes, each with radiation and a black hole in stable equilibrium. Each box serves as a heat reservoir, with its temperature determined by the mass of the black hole. Between the boxes there is a cylinder with a movable piston. The volume of the cylinder is much smaller than the volume of the reservoirs. The cylinder walls as well as the piston perfectly reflect the radiation. The cylinder can either be completely isolated, or can be open to one of the reservoirs. The pV diagram of the cycle is shown in Fig. 3.

The cycle starts with the piston close to the hot reservoir, and the volume of the working medium is zero.

(a) In the first step, the cylinder is open to the hot reservoir and the piston moves to increase the volume of the working medium to V_a. The process is isothermal, and, because the radiation pressure $p = aT^4/3$ is uniquely determined by the temperature, it is also isobaric. The work done by the system during this step is

$$W_a = p_1 V_a = \frac{1}{3} aT_1^4 V_a, \tag{13}$$

where p_1 and T_1 refer to the pressure and temperature of the hot reservoir. The energy extracted from the reservoir is

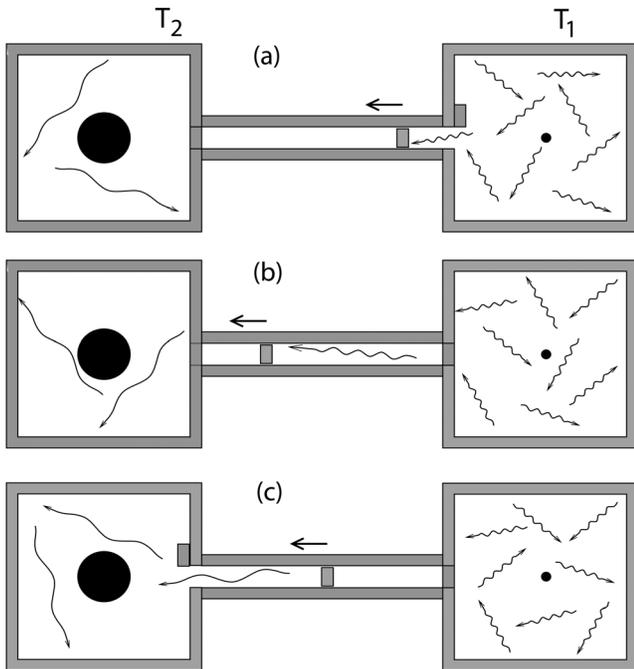$$Q_a = T_1 \Delta S_a = \frac{4}{3} aV_a T_1^4. \tag{14}$$

Fig. 2. A big black hole and cold radiation are in the left box and a small black hole and hot radiation are in the right box. (a) Radiation from the hot reservoir enters the cylinder and pushes the piston, so that the radiation in the cylinder expands isothermally. (b) The cylinder is isolated and the radiation expands adiabatically by pushing the piston and cooling to the temperature of the cold reservoir. (c) The cold radiation is pushed out of the cylinder isothermally into the cold reservoir.
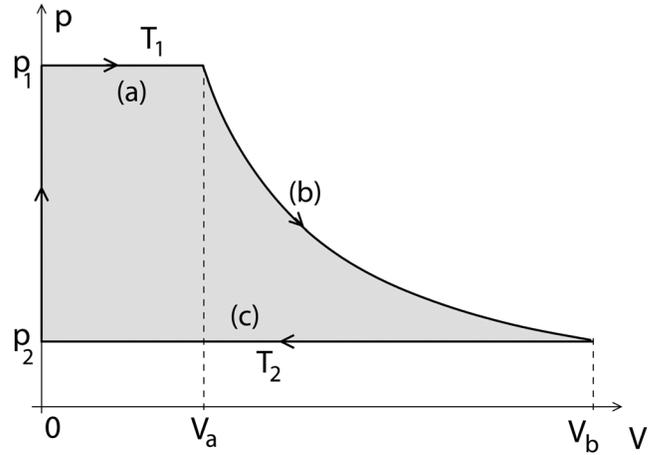
Fig. 3. A pV diagram of the cycle. The process consists of (a) isothermal (and isobaric) expansion, (b) adiabatic expansion, and (c) isothermal compression.

(b) In the second step, the cylinder is isolated and the radiation expands adiabatically by pushing the piston to volume V_b, cooling to the temperature T_2. Because the radiation entropy given by Eq. (7) remains constant, the resulting volume is $V_b = V_a(T_1/T_2)^3$, and the work done by the system in this step is

$$W_b = aT_1^4 V_a \left(1 - \frac{T_2}{T_1}\right). \tag{15}$$

(c) In the third step, the cylinder is opened to the cold reservoir and the radiation is isothermally pushed out. Because the radiation is pushed to the region with nonzero pressure, work must be performed on the system. The work is

$$W_c = -p_2 V_b = -\frac{1}{3} aT_1^3 T_2 V_a, \tag{16}$$

and the energy transported to the cold reservoir is

$$Q_c = T_2 \Delta S_c = \frac{4}{3} aV_b T_2^4 = \frac{4}{3} aV_a T_1^3 T_2. \tag{17}$$

(d) In the last step, the piston is carried through the empty cylinder to its initial position. No working medium is involved, so that no energy exchange or work occurs. The net work in one cycle is therefore,

$$W = W_a + W_b + W_c \tag{18a}$$

$$= \frac{4}{3} aV_a T_1^3 (T_1 - T_2) \tag{18b}$$

$$= Q_a - Q_c. \tag{18c}$$

As can be checked, the Carnot relations are satisfied,

$$\frac{Q_a}{Q_c} = \frac{T_1}{T_2} \tag{19}$$

with the efficiency η

$$\eta \equiv \frac{W}{Q_a} = 1 - \frac{T_2}{T_1}. \tag{20}$$

The radiation-like reservoirs and working fluids can be used for many other types of Carnot cycles, for example, for a cycle producing work in the form of maser radiation.[15]

IV. ENERGY EXTRACTION

Section III considered a single cycle during which the reservoirs were not significantly changed. However, because in each cycle energy is extracted from the hotter reservoir and deposited to the cooler, the properties of the reservoir change with time. This change can be expressed by Eq. (19) with $Q_a = -dM_1 c^2$ and $Q_c = dM_2 c^2$, which means that the energy transport is compensated by the decrease or increase of the black hole mass to find a new equilibrium with the radiation. Thus, we obtain

$$\frac{dM_1}{dM_2} = -\frac{T_1}{T_2}. \tag{21}$$

Because for black holes $T \propto 1/M$, we obtain

$$M_1 dM_1 = -M_2 dM_2, \tag{22}$$

with the solution

$$M_1^2 + M_2^2 = M_{1,0}^2 + M_{2,0}^2, \tag{23}$$

where $M_{1,0}$ and $M_{2,0}$ are the initial masses of the black holes. Note that due to the entropy relation (1), we can find this result directly by observing that during the reversible processes the total entropy does not change.

During this process the hotter reservoir loses mass and becomes hotter and the colder reservoir gains mass and becomes colder. This behavior is counterintuitive because typically the temperatures of two bodies in thermal contact converge. However, because a black hole has a negative heat capacity, its temperature increases while energy is being extracted.[13]

The black hole of the hot reservoir ultimately disappears, leaving the cold black hole with the final mass

$$M_f = \sqrt{M_{1,0}^2 + M_{2,0}^2}. \tag{24}$$

The total work extracted during the entire process is equivalent to the decrease of the mass,

$$W_{\text{tot}} = \left(M_{1,0} + M_{2,0} - \sqrt{M_{1,0}^2 + M_{2,0}^2} \right) c^2. \tag{25}$$

We illustrate this situation on the energy-entropy diagram in Fig. 4. The gray area represents the available states of a system consisting of black holes. The boundary $S \propto E^2$ represents states with a single black hole, that is, equilibrium states maximizing entropy for the given energy. Starting from nonequilibrium state A of several (two or more) black holes, equilibrium can be reached by many different paths. Path AB_1 corresponds to merging the black holes while keeping the energy fixed so that no work is extracted and the resulting state maximizes entropy with the available mass. Path AB_2 represents a collision of two black holes: a single black hole is produced and part of the energy is carried away by the gravitational waves. (According to Ref. 16 this energy is in the range of 10^{-3}–10^{-2} of the rest energy of the input black holes.) Path AB_4 corresponds to the reversible process
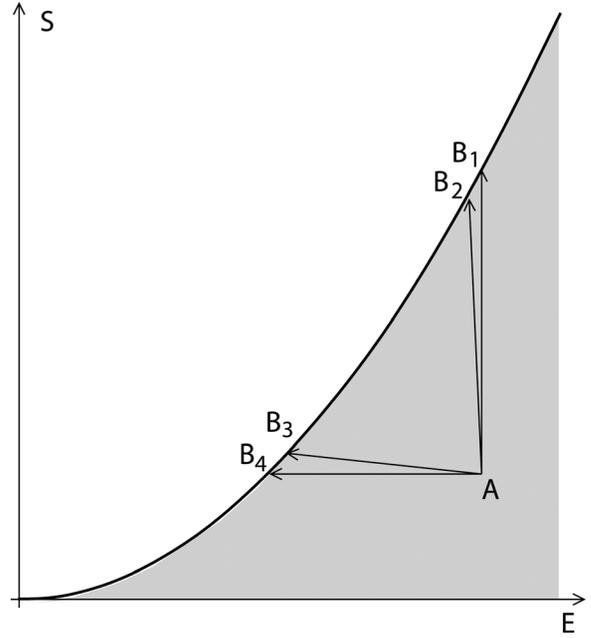

Fig. 4. Energy and entropy of a system of black holes. The gray area corresponds to nonequilibrium states with several black holes in the system, and the boundary $S \propto E^2$ represents equilibrium states with maximum entropy. A system in nonequilibrium state A can attain equilibrium by several paths. A difference in the E-coordinate represents the work gained from the system.

described in this section for which the entropy is unchanged and the maximum possible work is extracted. Path AB_3 represents a slightly more realistic version of this process, because due to imperfections and irreversibilities, the entropy increases and less work is gained.

V. POWER

The Carnot cycle is idealized because it assumes only the reversible processes, which are infinitely slow. In real processes a compromise must be reached between the requirements for the maximum efficiency and the maximum power. In the black hole process we have discussed, the power of the engine is limited by the rate at which the heat source can yield and the heat sink can absorb energy, which is given by the radiation power of the black holes (assuming that black holes emit only light[17]),

$$P_{\text{BH}} = 4\pi R_S^2 \sigma T^4 = \frac{\pi}{240} \frac{(kT)^2}{\hbar}, \tag{26}$$

where $\sigma = ac/4 = \pi^2 k^4/(60c^2\hbar^3)$ is the Stefan-Boltzmann constant. The power of the engine is the difference between the power of the heat source (energy released per second) and the power of the heat sink (energy absorbed per second),

$$P = P_H - P_C, \tag{27}$$

where the ratio of these powers is given by the Carnot relation

$$\frac{P_H}{P_C} = \frac{T_1}{T_2}. \tag{28}$$

To stay in thermal equilibrium, energy can only be released much slower from the hot reservoir than the black hole

radiates it, and the same holds also for the absorption of the cold reservoir. Therefore, we have the conditions

$$P_H \ll \frac{\pi k^2 T_1^2}{240\hbar}, \tag{29}$$

$$P_C \ll \frac{\pi k^2 T_2^2}{240\hbar}, \tag{30}$$

from which we find

$$P \ll \frac{\pi}{240} \frac{k^2 T_2 (T_1 - T_2)}{\hbar}. \tag{31}$$

To obtain a feeling for the numbers, we consider the colder black hole with the solar mass $M_2 = 2 \times 10^{30}$ kg, and $T_2 = 60$ nK, and the hotter one with the temperature of the solar center $T_1 = 1.6 \times 10^7$ K and $M_1 = 7.8 \times 10^{15}$ kg. The output power is smaller than 10^{-14} W. Although the energy available in this system is $\sim 7 \times 10^{32}$ J, it would take about 10^{39} yr, that is, 10^{29} times the age of the universe to extract it.

These estimates suggest that at present such an energy source would be useless in practice. However, at later stages of the universe, in a much colder environment when the flow of life would be at a much slower pace, methods like the one we have suggested might be a solution to the energy problem.

VI. EXTERNAL RADIATION

So far we have not considered the role of external radiation outside the boxes and the cylinder. If the universe outside the system is empty, we do not have to bother with pumping the waste radiation into the big black hole. Actually, the empty universe can be used as the cold reservoir at $T_2 = 0$, and we can use each black hole as a heat source and produce work by expanding their Hawking radiation. In the ideal case the extractable work is equal to the total energy of the black holes.

The situation is different in the presence of background radiation. Nevertheless, if the background radiation is colder than any of the black holes, it may still be more suitable to push the waste photons to the colder space rather than to the warmer black hole which would necessitate overcoming the higher pressure of the heat sink radiation. Moreover, compared with the black hole, space as the heat sink can absorb waste energy at an unlimited rate. The power of the engine is then only limited by the power of the hot black hole,

$$P \ll \frac{\pi}{240} \frac{k^2 T_1 (T_1 - T_2)}{\hbar}. \tag{32}$$

If a black hole is allowed to completely evaporate while running a Carnot engine, we can extract total work

$$W = M_0 c^2 \left(1 - \frac{T_2}{2 T_{1,0}} \right), \tag{33}$$

where M_0 is the initial black hole mass and $T_{1,0}$ is the corresponding temperature given by Eq. (4). Therefore, if the black hole and the background radiation start at the same temperature, we can still obtain $M_0 c^2 / 2$ of useful work by vaporizing the black hole. Even if the black hole is colder than the background radiation (but not colder than $T_2/2$) and

we have to initially invest some work to pump the energy out of it, we can obtain positive net work.

If the space is filled with thermal background radiation hotter than the black holes, we can use the radiation as the hot reservoir and deposit the waste energy into the black holes. This method resembles the Geroch model discussed by Bekenstein[3] and studied in more detail by Deng and Gao,[11] who proposed its reversible version. Their model assumes lowering a box filled with thermal radiation to the vicinity of the black hole horizon and releasing its content there.

In this case the power is limited by the rate with which the hole can absorb radiation at the same temperature as the hole itself. Thus, if the radiation has temperature T_1 and the black hole has temperature T_2, the power is limited by Eq. (31). For $T_2 \ll T_1$, the power is proportional to the black hole temperature, $P \propto T_2 \propto 1/M$. Because the rate of mass increase is limited by $T_2^2 \propto M^{-2}$, we can solve the relation $dM/dt \propto 1/M^2$ as $M \propto t^{1/3}$, which leads to $P \propto t^{-1/3}$. Thus, the work increases with time because $W \propto t^{2/3}$. Therefore, the extractable work would be unlimited, provided that the background radiation stays at a constant temperature.

VII. BLACK HOLE ECONOMY

If we have two black holes, what would be the best way to use them? We could combine them to gain useful work and create a bigger hole. Or, we could evaporate both of them to accomplish some work and produce radiation. The latter option would perform the work faster, but in the end there would be no black hole left. Taking into account that thermal radiation is the ultimate waste, the former option is environmentally friendlier. In a sense, it means gaining energy by cleaning the mess out of the universe and concentrating it in the black hole.

For longer term investments, we might prefer growing black holes as condensed energy supplies. While growing the holes, we can also extract useful work, but at much slower pace than with evaporating the holes. But, provided that the expansion of the universe continues, and the background radiation cools down, all the black holes will ultimately be hotter than the background radiation and no more work can be obtained by feeding the background radiation to the holes. Merging black holes or getting energy out of them and radiating the waste energy to empty space would be the only source of useful energy available. A reasonable strategy could be having a portfolio of black holes of various sizes, which would give us the option of using small holes for fast energy extraction and big holes as large energy supplies for later use. Careless individuals would collide their black holes to produce gravitational waves for fun or let small holes explode as fireworks, which would be wasting precious resources. Rentiers who like regular energy income might prefer having their N holes with masses distributed as $n^{1/3}$, with $n = 1, 2, ..., N$ (following from the black hole evaporation lifetime $\propto M^3$).

The black hole economy is expected to end within about 10^{106} yr when even the galaxy size black holes will evaporate.[1]

VIII. DISCUSSION AND CONCLUSION

Previous models that used black holes as a heat sink of heat engines[3,11] moved a box with radiation to the black hole

horizon where the radiation was released. This model could cause some difficulties for students seeking to understand the essential parts of the model and raises questions such as: How can we control the box near the horizon if time there flows much more slowly? How can the radiation in a small box near a big hole be brought to thermal equilibrium with the hole if the wavelength of such radiation is comparable to the Schwarzschild diameter? Our approach is based on building heat reservoirs as combined systems of a black hole and radiation in isolated boxes, which leads to a straightforward heat exchange between the reservoirs and the engine.

In our method, the reservoir has simple properties provided that the size of the box is suitably chosen; that is, it should be sufficiently small so that the heat capacity is dominated by that of the black hole and sufficiently large so that effects such as gravitational red shifts can be neglected at the box opening. The heat exchange between the reservoir and the engines must be sufficiently slow so that the reservoir has time to reach thermal equilibrium via the interaction of the black hole with the field. The size condition can be satisfied if the mass of the black hole is much larger than the Planck mass, which holds for most of the lifetime of the black hole. After a long operation of the Carnot engine the size requirements could change. For example, the black hole inside the hot reservoir would become so small that a smaller box is necessary for the system to remain stable (see Fig. 1 and follow the lines from top to bottom). This adjustment could be done without changing the basic properties of the model, namely, not affecting the reversibility. We could, for example, insert a partition into the box, leaving the black hole in one part with the other part containing only the radiation. The radiation can expand adiabatically, doing work and decreasing the temperature to that of the cold reservoir into which it will be finally deposited. The part of the box with the black hole is then used as the size-adjusted reservoir. This simple method breaks down only at the terminal stage when the mass of the hot black hole approaches the Planck mass, and the stability condition requires such a small box that the gravitational effects at the box walls would become important.

The model assumes various problems can be solved. For example, the boxes should keep the radiation under adiabatic conditions for times longer than the present age of the universe and not thermalize themselves (because such thermalization would imply that the outer wall of the box would be at the same temperature as the interior, and the system would loose energy by thermal radiation). Their walls should reflect perfectly all the radiation emitted from the black hole: that is, not only the electromagnetic field but also neutrinos and gravitons that are natural parts of Hawking radiation.[12] The walls must resist extreme pressures if the mass of the black hole becomes relatively small. In the example of Sec. V the black hole of mass $\sim 10^{15}$ kg was kept in a container with radiation whose pressure is comparable to that of the solar core. Even the most sophisticated material made of atoms would fail if protons are unstable on time scales required for the processes—perhaps 10^{34} yr.[1] More sophisticated kinds of materials would be needed in such a case.

Another problem would be keeping the black holes separated, resisting their gravitational attraction. A system in which black holes orbit around each other would radiate gravitational waves, and thus loose energy. In principle, the gravitational waves can be collected as useful work which could be reused for keeping the system in their proper orbits. These problems are far beyond the scope of this paper, and we happily leave them for future generations.

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[3] J. D. Bekenstein, "Black holes and entropy," Phys. Rev. D 7, 2333–2346 (1973).

[4] A simple estimate is based on the assumption that a person consumes about 2000 kcal $\approx 10^7$ J in food per day for about 75 yr and dissipates most of it as heat at ≈ 300 K.

[5] This estimate is based on the incoming power 10^3 W of solar radiation per square meter multiplied by the illuminated area of $\approx 1.3 \times 10^{14}$ m^2. The age of the Earth is $\approx 5 \times 10^9$ yr so that the total received energy is about 2×10^{34} J. The same amount of energy is released as heat at the Earth's surface temperature of ≈ 300 K, yielding the entropy increase of $\approx 10^{32}$ J/K. The ratio of outcoming/incoming numbers of photons is equal to the ratio of the solar and Earth temperatures ($\approx 5000/300 \approx 17$, using the same numbers as in Ref. 1). As stressed in Ref. 1, the dominant parameter characterizing the entropy is the number of particles carrying energy, because the entropy is linearly proportional to this number and depends only logarithmically on other parameters.

[6] The estimate is based on the solar power of $\approx 4 \times 10^{26}$ W multiplied by its age of $\approx 5 \times 10^9$ yr, corresponding to $\approx 5 \times 10^{43}$ J of released energy. If we divide it by the solar surface temperature $\approx 5 \times 10^3$ K, we obtain the entropy estimate of $\approx 10^{40}$ J/K. The number of created photons follows from the ratio of released nuclear energy per particle ≈ 7 MeV to the energy of a visible photon ≈ 1 eV, corresponding to the solar surface temperature.

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[17] The assumption that the black hole emits only photons is made to simplify the calculations; in reality a temperature-dependent factor in Eq. (26) would reflect various fields that are produced during black hole evaporation. See, for example, Ref. 12.