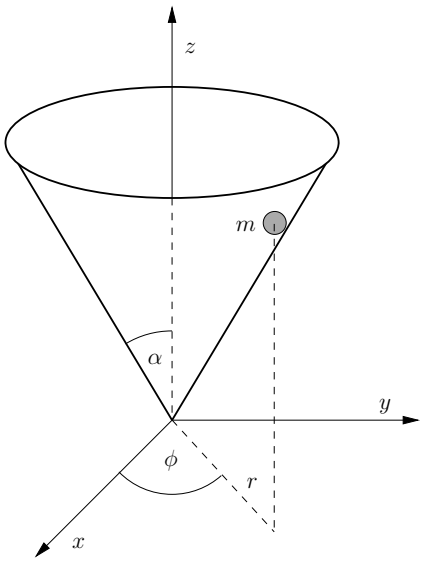


Zylinderkoordinaten:

$$T-U=\frac{m}{2}\left(\dot{r}^2+\dot{z}^2+r^2\dot{\phi}^2\right)-mgz$$

Zwangsbedingung: $z=r\cot\alpha$

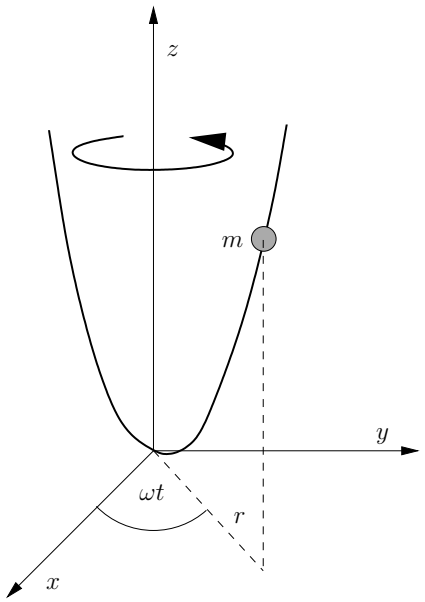
$$L=\frac{m}{2}\left[(1+\cot^2\alpha)\dot{r}^2+r^2\dot{\phi}^2\right]-mgr\cot\alpha$$

Lagrange-Gleichung für r:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{r}}-\frac{\partial L}{\partial r}=m\left[(1+\cot^2\alpha)\ddot{r}-r\dot{\phi}^2+g\cot\alpha\right]=0$$

Lagrange-Gleichung für ϕ:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{\phi}}-\frac{\partial L}{\partial\phi}=mr(2\dot{r}\dot{\phi}+r\ddot{\phi})=0\,.$$



Zylinderkoordinaten:

$$T-U=\frac{m}{2}\left(\dot{r}^2+\dot{z}^2+r^2\omega^2\right)-mgz$$

Form des Drahtes: $z=f(r)$

$$L=\frac{m}{2}\left(\dot{r}^2+(f'(r)\dot{r})^2+r^2\omega^2\right)-mg\,f(r)$$

Lagrange-Gleichung für r:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{r}}-\frac{\partial L}{\partial r}=m\left[1+f'(r)^2\right]\ddot{r}+m\,f'(r)\,f''(r)\,\dot{r}^2+mg\,f'(r)-m\omega^2 r=0\,.$$