Electrical Engineering

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Exercise Booklet for DC and AC Circuits

Content

This task booklet was created with LaTeX and pdfTeX using the following packages:

`lmodern`: Font

Ti*k*Z: Drawings

PGFPLOTS: Plots

CircuiTi*k*Z: Circuit diagrams

siunitx: Units

Task 1: Quantities and units ★☆☆

Calculate the time t in which 1 L of water is warmed up by a heater with the power $P = 2\,\text{kW}$ from the temperature $\vartheta_1 = 20\,°\text{C}$ to $\vartheta_2 = 70\,°\text{C}$. The heat dissipation to the environment is negligible.

Note: The specific heat capacity of water is $c = 4187\,\frac{\text{W s}}{\text{kg K}}$.

Task 2: Calculating in decibels ★⯪☆

For an interference voltage V_2, a level of $L_{V2} = 110\,\text{dB}$ is specified. This value refers to $V_1 = 1\,\mu\text{V}$. How large is the voltage V_2 in V?

Note: Level in dB $= 20 \cdot \lg\left(\frac{V_2}{V_1}\right)$

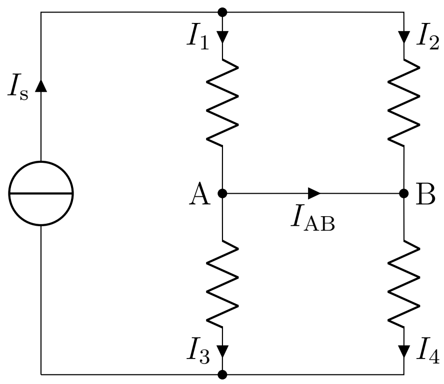
Task 3: Efficiency ★☆☆

A pump is to lift $20\,\text{m}^3$ of water per hour into a container that is 25 m higher. The efficiency of this pump is $\eta_\text{P} = 70\,\%$.

What electrical power must the drive motor absorb at a motor efficiency of $\eta_\text{M} = 90\,\%$?

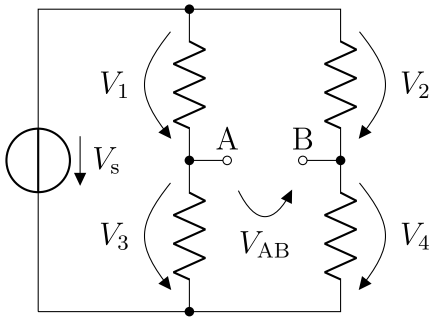
Task 4: Kirchhoff current law ⯪☆☆

From $I_1 = 2\,\text{A}$, $I_2 = 3\,\text{A}$ and $I_4 = 4\,\text{A}$ calculate I_s, I_3 and I_AB with the correct sign.



Task 5: Kirchhoff's voltage law ★☆☆

From $V_1 = 2\,\mathrm{V}$, $V_2 = 3\,\mathrm{V}$ and $V_4 = 5\,\mathrm{V}$ calculate V_s, V_3 and V_AB with the correct sign.



Task 6: Current-voltage characteristic ★☆☆

Along the series connection of 5 components (resistors and voltage sources) of unknown size and sequence, the following potentials were measured against a reference potential $\varphi_\mathrm{ref} = 0$ at the connection points A to F:

an Punkt	A	B	C	D	E	F
Potential in V; Short circuit between A and F	−10	14	8	−4	−6	−10
Potential in V; 6 Ω between A and F	−10	14	11	−1	−2	−4

Calculate the values of the components and the voltage to be expected at terminals A and F in open circuit.

Task 7: Wire length ★☆☆

A long copper wire with a specific conductivity of $58 \times 10^6\,\frac{\mathrm{S}}{\mathrm{m}}$ is wound onto a reel. To determine the length of the wire without unwinding it, its resistance is measured. This is $5.747\,\Omega$. The wire has a cross-sectional area of $0.75\,\mathrm{mm}^2$.

How long is the wire?

Task 8: Wire material ★★☆

An underground cable is $10\,\mathrm{km}$ long and has a conductor cross-section of $120\,\mathrm{mm}^2$. The cable has a resistance of $2.25\,\Omega$.

What is the specific conductivity of the conductor material?

What material is the wire probably made of?
(see `https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity`)

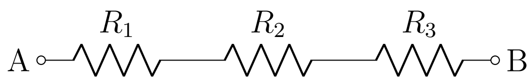
Task 9: Resistance ★ ☆ ☆

A voltage drop of $V = 2\,\text{V}$ is measured across an aluminum conductor with a cross-section area of $A = 2.5\,\text{mm}^2$ and a length of $l = 18\,\text{m}$. The specific conductivity of aluminum is $\gamma = 36\,\frac{\text{m}}{\Omega\,\text{mm}^2}$.

What is the current density J and the current amplitude I inside the conductor?

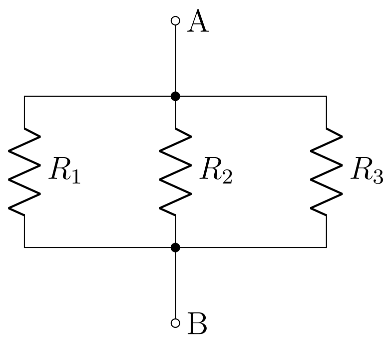
Task 10: Series connection ★ ☆ ☆

Three resistors $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$ and $R_3 = 30\,\Omega$ are connected in series. What is the equivalent resistance between terminals A and B?



Task 11: Parallel connection ★ ☆ ☆

Three resistors $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$ and $R_3 = 30\,\Omega$ are connected in parallel. What is the equivalent resistance between terminals A and B?



Task 12: Series and parallel connection ★ ★ ☆

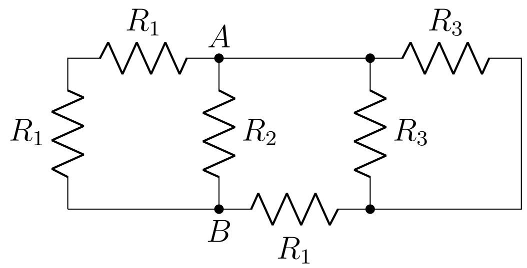
With two resistors R_1 and R_2 individually and in combination, four resistance levels can be realized e. g. for a hotplate.

Calculate the ratio $\frac{R_1}{R_2}$ so that the same resistance ratios result from level to level.

Task 13: Equivalent resistance ★ ★ ☆

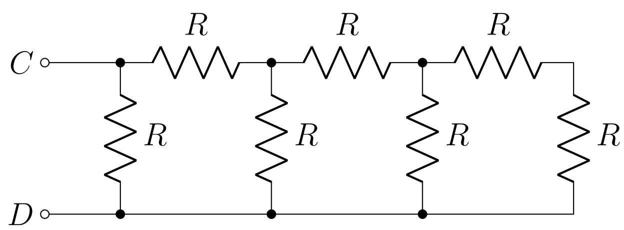
Calculate the equivalent resistances

a) R_{AB},



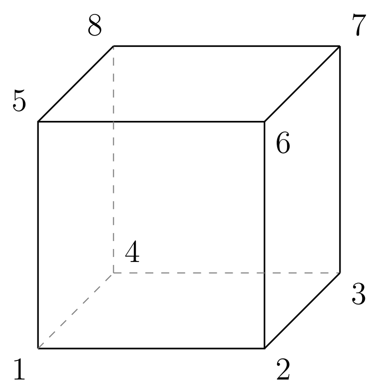
$R_1 = 1\,\Omega$ $\quad$ $R_2 = 2\,\Omega$ $\quad$ $R_3 = 6\,\Omega$

b) R_{CD}, expressed in terms of R,



Task 14: Equivalent resistance ★ ★ ★

Twelve resistors of $1\,\mathrm{k\Omega}$ each form the edges of a cube.

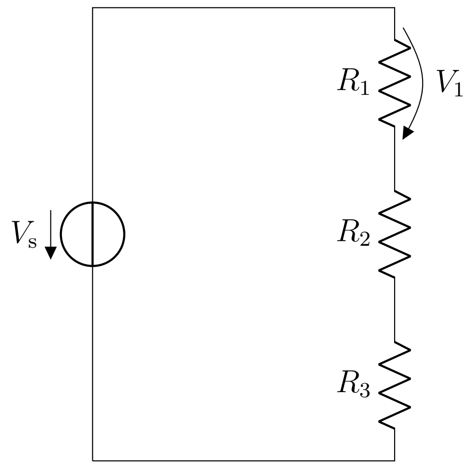


How large is the equivalent resistance between the corner points 1 and 7.

Note: Use symmetry and potential equality!

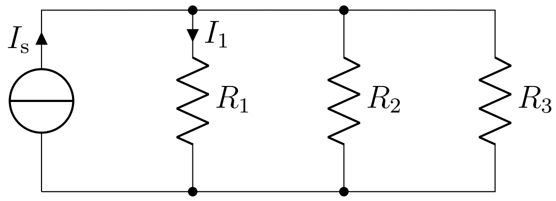
Task 15: Voltage divider rule ★☆☆

Three resistors $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$ and $R_3 = 30\,\Omega$ are connected in series to a voltage source with a source voltage of $V_s = 9\,\text{V}$. What is the partial voltage V_1 across the resistor R_1?



Task 16: Current divider rule ★☆☆

Three resistors $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$ and $R_3 = 30\,\Omega$ are connected in parallel to a current source with a source current of $I_s = 12\,\text{A}$. What is the partial current I_1 through the resistor R_1?

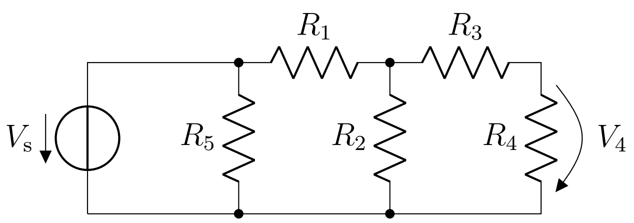


Task 17: Voltage and current divider rule ★★☆

Calculate the voltage V_4 using the

a) voltage divider rule,

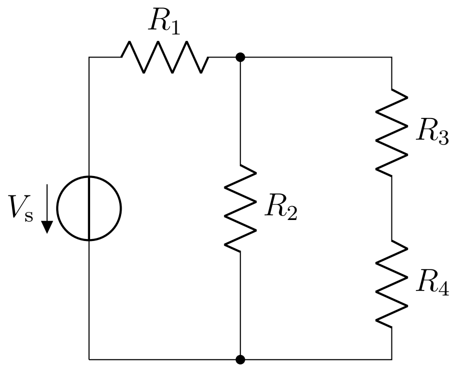
b) current divider rule.



$R_1 = R_3 = R_4 = R_5 = 1\,\Omega$ $\qquad R_2 = 2\,\Omega$ $\qquad V_s = 4\,\text{V}$

Task 18: Electrical power ★★☆

Which resistor is the one with the highest power loss? How large is it?



$V_s = 100\,\text{V}$ $R_1 = 30\,\Omega$ $R_2 = 210\,\Omega$ $R_3 = 30\,\Omega$ $R_4 = 75\,\Omega$

Task 19: Maximum power transfer theorem ★½☆

A rechargeable battery is regarded as a voltage source with an internal series resistance. When discharging the battery at a $0.5\,\Omega$ load, a terminal voltage of $3\,\text{V}$ and a current of $6\,\text{A}$ are observed. When charging the battery, a terminal voltage of $9\,\text{V}$ and a current of $-3\,\text{A}$ are measured.

What is the maximum power the battery can deliver?

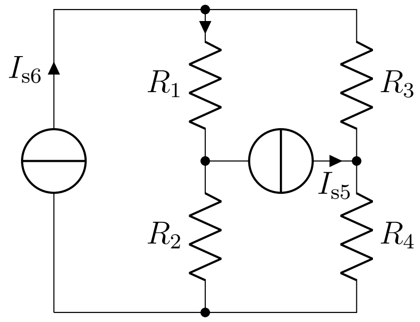
Task 20: Circuit analysis ★★☆

Calculate the current I_1 in the following circuit

a) by superposition,

b) with the help of branch current analysis

c) using nodal voltage analysis.



$R_1 = 10\,\Omega$ $R_2 = 20\,\Omega$ $R_3 = 30\,\Omega$ $R_4 = 40\,\Omega$ $I_{s5} = 5\,\text{A}$ $I_{s6} = 6\,\text{A}$

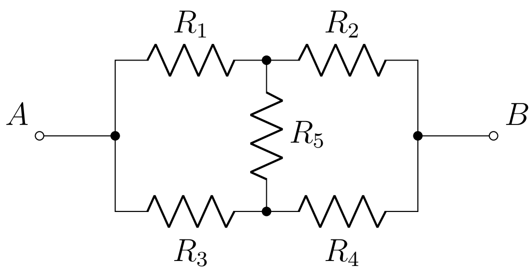
Task 21: Star-delta transformation ★ ★ ☆

Calculate the resistance R_{AB}

a) using a delta-star transformation,

b) with the help of a star-delta transformation,

c) using nodal voltage analysis.



$R_1 = 1\,\Omega$ $\quad$ $R_2 = 2\,\Omega$ $\quad$ $R_3 = 3\,\Omega$ $\quad$ $R_4 = 4\,\Omega$ $\quad$ $R_5 = 5\,\Omega$

Task 22: Plate capacitor ★ ⯪ ☆

Two thin square metal plates with a side length of $a = 10\,\text{cm}$ each are arranged parallel to each other at a distance of $d = 1\,\text{mm}$ and form a plate capacitor. Between the plates is air with a permittivity of $\varepsilon = 8.854 \times 10^{-12}\,\frac{\text{A s}}{\text{V m}}$.

a) What is the capacitance C of the plate capacitor?

b) The plate capacitor is charged to a DC voltage of $V = 10\,\text{V}$. How large is the charge Q that is stored on the plate capacitor?

c) The charged plate capacitor is disconnected from the DC voltage source. The plate spacing is then increased to a value of $d = 5\,\text{mm}$. To what value does the capacitance C of the plate capacitor change? What does this mean for the voltage V between the two plates?

d) The charged plate capacitor is now discharged with a constant current of $I = 1\,\text{mA}$. What is the rate of voltage change ${}^{\Delta V}/_{\Delta t}$? How long does it take until the plate capacitor is completely discharged (to a voltage of $V = 0\,\text{V}$)?

Task 23: Inductance ★ ⯪ ☆

A coil with an inductance of $L = 1\,\text{mH}$ and a copper resistance of $R = 1\,\Omega$ is connected to a voltage source with a constant source voltage of $V = 10\,\text{V}$.

a) What is the rate of change of current ${}^{\mathrm{d}i}/_{\mathrm{d}t}$ immediately after connection?

b) What is the constant direct current I to which the coil is finally charged?

c) What is the magnetic flux Φ through the coil?

Task 24: Equivalent capacitance ★ ☆ ☆

What is the equivalent or total capacitance C_{tot} of two capacitors with a respective capacitance of $C = 1\,\mu\text{F}$ if they are connected

a) in series

b) or parallel?

Task 25: Equivalent inductance ★ ☆ ☆

What is the equivalent or total inductance L_{tot} of two coils with a respective inductance of $L = 1\,\text{mH}$ if they are connected

a) in series

b) or parallel?

Task 26: Complex phasor ★ ☆ ☆

Given is a voltage as a time function

$$v(t) = \hat{v} \cdot \cos(\omega t + \varphi_v) = 5\,\text{V} \cdot \cos(\omega t + 60°)\,.$$

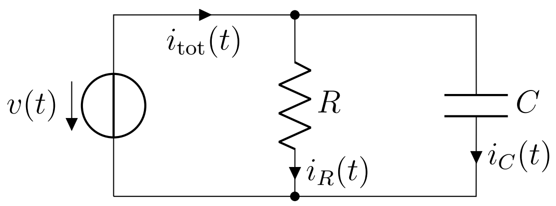
This time function is to be converted into the phasor form (stationary phasor) in:

- exponential form
- trigonometric form
- Cartesian form

The phasor is to be drawn in the complex plane.

Task 27: Impedance and admittance ★ ★ ☆

An ohmic-capacitive load is connected to an ideal voltage source.



$$v(t) = \sqrt{2} \cdot 230\,\text{V} \cdot \sin(\omega t + 30°) \qquad R = \frac{1}{\omega C} = 23\,\Omega$$

Calculate

a) the complex total admittance $\underline{Y}$ and impedance $\underline{Z}$ of the load,

b) the currents $i_R(t)$, $i_C(t)$ and $i_{\text{tot}}(t)$ using the voltage,

c) the total current $i_{\text{tot}}(t)$ by adding $i_R(t)$ and $i_C(t)$,

d) the currents $i_R(t)$ and $i_C(t)$ using the current divider rule.

using complex quantities and draw all phasors in the complex diagram.

Task 28: Complex power ★★☆

A resistor with a resistance $R = 10\,\Omega$ and an inductor with an inductance of $L = 31.83\,\text{mH}$ are connected in series to an ideal voltage source with a sinusoidal voltage of $v_\text{s}(t) = 325\,\text{V} \cdot \sin(\omega t + 30°)$ with a frequency of $f = 50\,\text{Hz}$.

a) Calculate the active power P dissipated in the resistor R, the (inductive) reactive power Q_L at the inductance L and the apparent power S at the series connection of R and L.

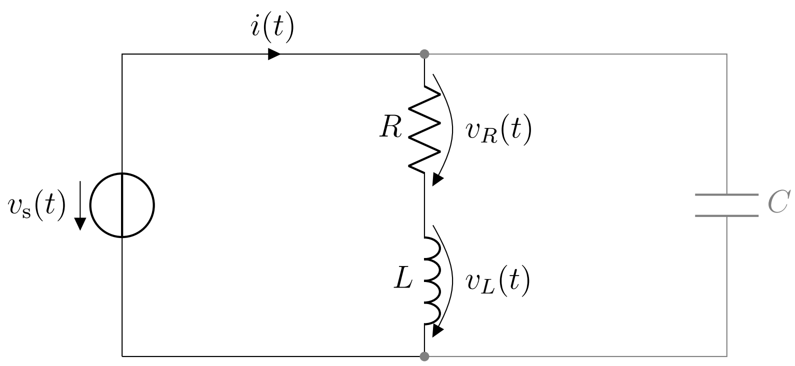
b) Calculate the complex power $\underline{S}$ at the series connection of R and L, its real part $\Re\{\underline{S}\}$, imaginary part $\Im\{\underline{S}\}$ and magnitude $|\underline{S}|$. Compare these values with the previously calculated active power P, reactive power Q_L and apparent power S.

c) How large is the (inductive) power factor $\cos\varphi$?

d) How large must the reactance X_C or the impedance $\underline{Z}_C$ of a capacitance C connected in parallel be so that the reactive power is completely compensated for in the overall circuit? What is the active power and apparent power of the overall circuit?

e) What is the required capacitance C?

f) Draw the corresponding power triangle.



Task 29: Three-phase star-star system ★ ⯪ ☆

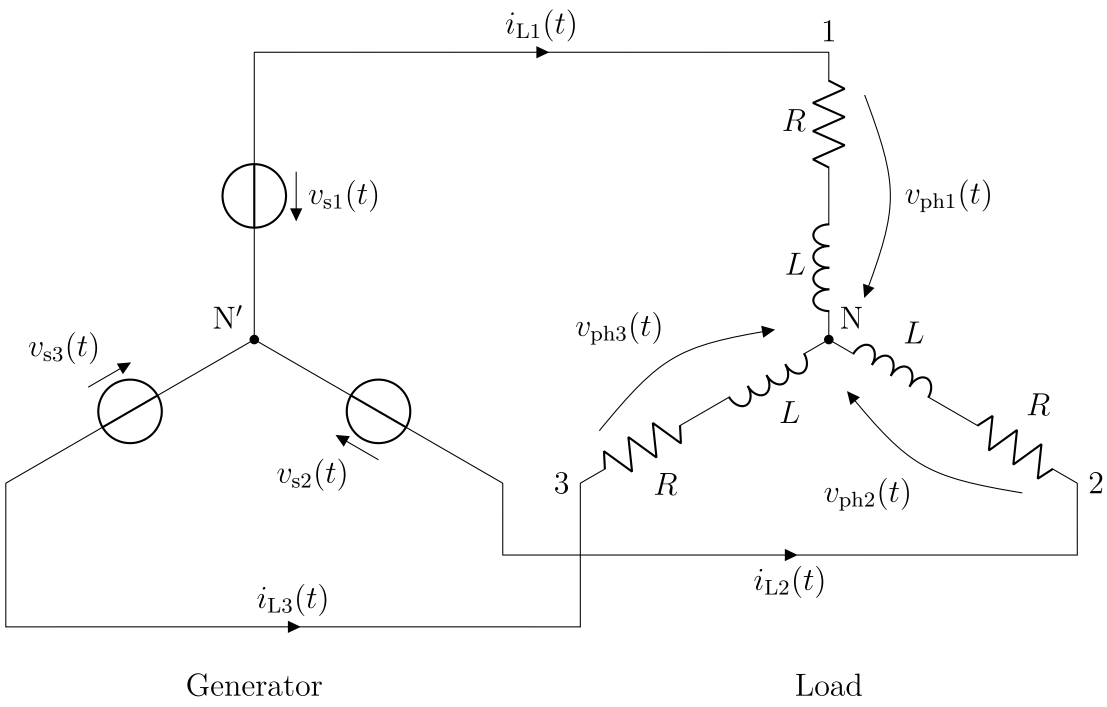
A symmetrical generator and a symmetrical load in a star connection are given.

a) Calculate the branch voltages of the load and the phase-to-phase voltages by magnitude and phase and draw the phasor diagram of all voltages. The root-mean-square values shall be given.

b) Calculate the branch currents of the load and the phase currents by magnitude and phase. The root-mean-square values should also be given here.

c) What would be the current in the neutral conductor if N′ and N were connected (calculation)?

d) Calculate the active power P, the reactive power Q and the apparent power S of the load.



$$v_{s1}(t) = \hat{v} \cdot \sin(\omega t) \qquad v_{s2}(t) = \hat{v} \cdot \sin(\omega t - 120°) \qquad v_{s3}(t) = \hat{v} \cdot \sin(\omega t - 240°)$$

$$\hat{v} = \sqrt{2} \cdot 230\,\text{V} \qquad R = \omega L = 10\,\Omega$$

Task 30: Three-phase star-delta system ★ ★ ☆

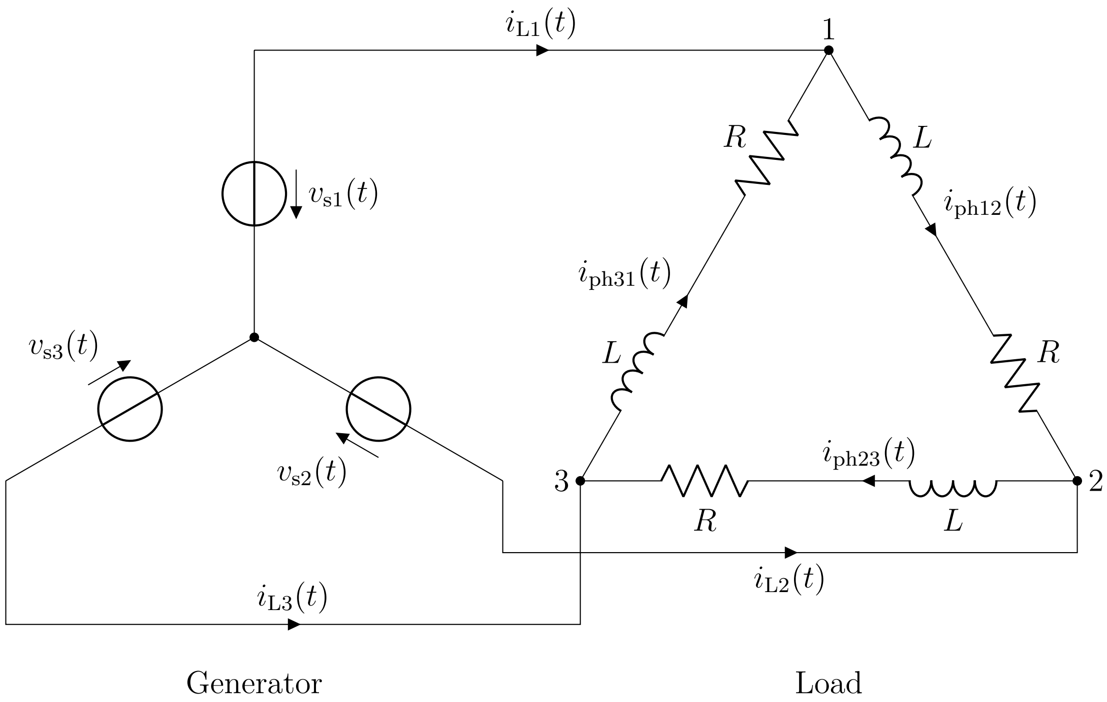
A symmetrical generator in a star connection and a symmetrical load in a delta connection are given.

a) Calculate the branch voltages of the load and the phase-to-phase voltages by magnitude and phase and draw the phasor diagram of all voltages. The root-mean-square values shall be given.

b) Calculate the branch currents of the load and the phase currents by magnitude and phase and draw the phasor diagram of all currents. The root-mean-square values should also be given here.

c) Calculate the active power P, the reactive power Q and the apparent power S of the load.

d) Compare the powers with those from task 29.

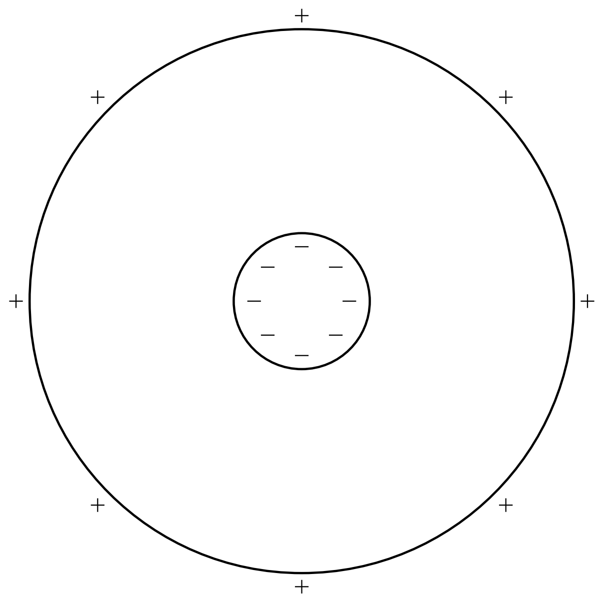


$v_{s1}(t) = \hat{v} \cdot \sin(\omega t)$ $\quad v_{s2}(t) = \hat{v} \cdot \sin(\omega t - 120°)$ $\quad v_{s3}(t) = \hat{v} \cdot \sin(\omega t - 240°)$

$\hat{v} = \sqrt{2} \cdot 230\,\text{V}$ $\quad R = \omega L = 10\,\Omega$

Task 31: Coaxial cable ★ ☆ ☆

A coaxial cable is given. The charge $-Q$ is on the inner conductor and the charge $+Q$ is on the outer conductor.



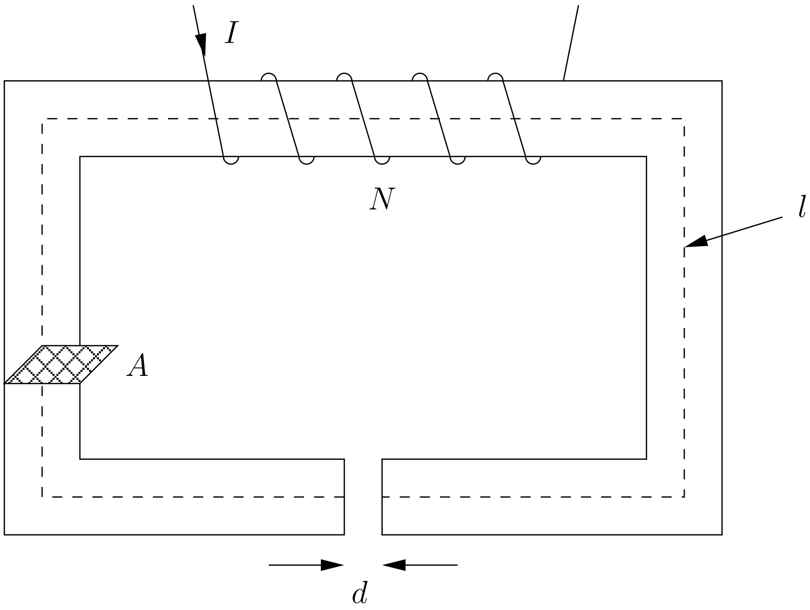
Add the field lines and equipotential lines of the electric field strength to the drawing.

Task 32: Point charges ★ ☆ ☆

Sketch the electric field lines and equipotential lines of the following arrangement of two charges.

Task 33: Magnetic circuit ★ ★ ☆

Given is a magnetic circuit with the mean iron length ℓ and the cross-sectional area A. An air gap of thickness d is cut into the iron core. A coil with N turns is wound around the iron core, through which the direct current I flows. The material of the iron core has the relative permeability μ_r.



a) Draw the equivalent electrical circuit diagram of the magnetic circuit.

b) Calculate the values of all electrical components present.

c) Calculate the magnetic flux Φ in the iron core.

d) Calculate the magnetic field strength H in the air gap.

Task 34: Charging and discharging of a capacitor ★★☆

An initially empty capacitor with a capacitance of $C = 10\,\mu\text{F}$ is charged via a charging resistor $R_\text{c} = 0.8\,\text{M}\Omega$ by a DC voltage source $V_\text{s} = 500\,\text{V}$.

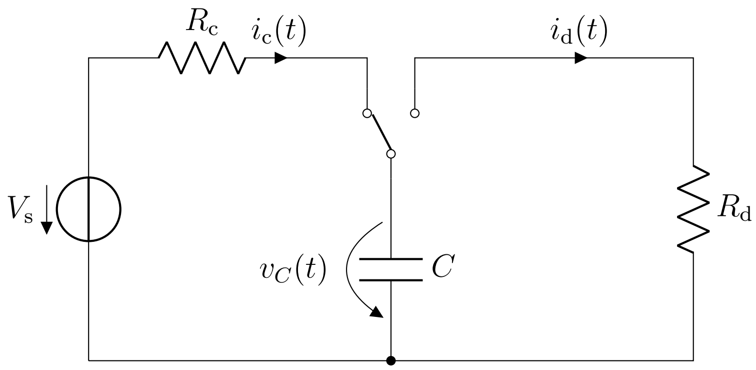
a) Calculate the time course of the voltage $v_C(t)$ during charging.

b) What is the maximum charging current?

c) After what time is the charging practically finished?

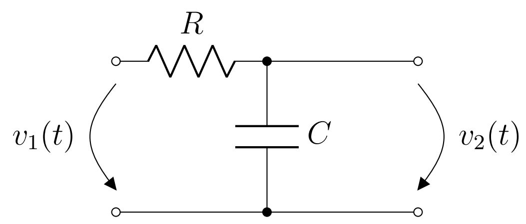
d) What is the maximum discharge current if the capacitor is discharged via a discharging resistor $R_\text{d} = 0.5\,\Omega$ after charging?

e) What is the energy dissipation in the discharging resistor R_d during discharge?



Task 35: Filter ★ ⯪ ☆

A resistor with $R = 10\,\Omega$ and a capacitor with $C = 318.3\,\mu\mathrm{F}$ are connected in series. A sinusoidal input voltage $v_1(t)$ is applied to this series connection. The output voltage $v_2(t)$, which is also sinusoidal, is measured across the capacitor.



a) Use the voltage divider rule to determine the complex voltage transfer ratio $\underline{\hat{v}}_2/\underline{\hat{v}}_1$ as a formula.

b) How large is this voltage transfer ratio $\underline{\hat{v}}_2/\underline{\hat{v}}_1$ in magnitude and phase for the following frequencies (approximately)?

- $5\,\mathrm{Hz}$
- $50\,\mathrm{Hz}$
- $500\,\mathrm{Hz}$

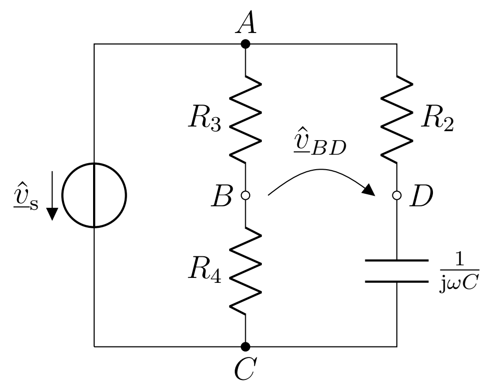
c) What function does this circuit fulfill? What kind of filter is it?

Task 36: Hausrath bridge ★ ★ ☆

For the AC bridge circuit shown (Hausrath bridge), determine the bridge voltage $\underline{\hat{v}}_{BD}$ in general according to magnitude and phase if $R_3 = R_4$.

In what way does the voltage $\underline{\hat{v}}_{BD}$ depend on the resistance $R_2 = 0 \ldots \infty$?

Qualitatively draw the phasor image of the voltages in the complex plane.



Result check

1. $t = 104.7\,\mathrm{s}$
2. $V_2 = 0.316\,\mathrm{V}$
3. $P_{\mathrm{el}} = 2.16\,\mathrm{kW}$
4. $I_{\mathrm{s}} = 5\,\mathrm{A}$; $I_3 = 1\,\mathrm{A}$; $I_{\mathrm{AB}} = 1\,\mathrm{A}$
5. $V_{\mathrm{s}} = 8\,\mathrm{V}$; $V_3 = 6\,\mathrm{V}$; $V_{\mathrm{AB}} = 1\,\mathrm{V}$
6. $V_{\mathrm{AB}} = -24\,\mathrm{V}$; $R_{\mathrm{BC}} = 3\,\Omega$; $V_{\mathrm{CD}} = 12\,\mathrm{V}$; $R_{\mathrm{DE}} = 1\,\Omega$; $R_{\mathrm{EF}} = 2\,\Omega$; $V_{\mathrm{FA,open}} = 12\,\mathrm{V}$
7. $l = 250\,\mathrm{m}$
8. $\gamma = 37 \times 10^6\,\frac{\mathrm{S}}{\mathrm{m}}$ (aluminum)
9. $I = 10\,\mathrm{A}$; $J = 4\,\frac{\mathrm{A}}{\mathrm{mm}^2}$
10. $R_{\mathrm{AB}} = 60\,\Omega$
11. $R_{\mathrm{AB}} = 5.45\,\Omega$
12. $\frac{R1}{R_2} = 0.618$ (if $R_1 < R_2$) or $\frac{R1}{R_2} = 1.618$ (if $R_1 > R_2$)
13. a) $R_{AB} = 0.8\,\Omega$
    b) $R_{CD} = 0.619 \cdot R$
    c) $R_{EF} = 2\,\Omega = R_{\mathrm{w}}$
14. $R_{17} = 833\,\Omega$
15.
16.
17. $V_4 = 1\,\mathrm{V}$
18. $P_4 = 33.33\,\mathrm{W}$
19. $P_{\mathrm{a,max}} = 18.375\,\mathrm{W}$
20. $I_1 = \dfrac{(R_2 + R_4) \cdot I_{\mathrm{q5}} + (R_3 + R_4) \cdot I_{\mathrm{q6}}}{R_1 + R_2 + R_3 + R_4}$
    $I_1 = 7.2\,\mathrm{A}$; $I_2 = 2.2\,\mathrm{A}$; $I_3 = -1.2\,\mathrm{A}$; $I_4 = 3.8\,\mathrm{A}$
21. $R_{\mathrm{AB}} = 2.094\,\Omega$
22.
23.
24.
25.
26. -
27. a) $\underline{Z} = 16.26\,\Omega \cdot \mathrm{e}^{-\mathrm{j}\,\cdot\,45°}$

b) $i_R(t) = 14.14\,\mathrm{A} \cdot \sin(\omega t + 30°)$; $i_C(t) = 14.14\,\mathrm{A} \cdot \sin(\omega t + 120°)$; $i_{\mathrm{tot}}(t) = 20\,\mathrm{A} \cdot \sin(\omega t + 75°)$

c) -

d) -

28. a) -

b) -

c) -

d) $v_{\mathrm{br}}(t) = 6.5\,\mathrm{V} \cdot \sin(\omega t + 113.13°)$

29. a) -

b) -

c) $i_{\mathrm{N}} = 0$

d) $P = 7932\,\mathrm{W}$; $Q = 7932\,\mathrm{var}$; $S = 11\,220\,\mathrm{VA}$

e) -

30. a) -

b) -

c) $P = 24.0\,\mathrm{kW}$; $Q = 24.0\,\mathrm{kvar}$; $S = 33.9\,\mathrm{kVA}$

d) -

31.

32.

33.

34. a) $v_C(t) = V_{\mathrm{s}} \cdot \left(1 - \mathrm{e}^{-\frac{t}{R_{\mathrm{C}}C}}\right)$

b) $i_{\mathrm{c,max}} = 0.625\,\mathrm{mA}$

c) $24\,\mathrm{s}$

d) $i_{\mathrm{d,max}} = 1000\,\mathrm{A}$

e) $E = 1.25\,\mathrm{W\,s}$