CONSERVATION LAWS and applicability OF THE EQUIVALENCE OF INERTIAL FRAMES: the Doppler RADAR

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1 author:

Stefano Quattrini
Ordine degli ingegneri della provincia di Ancona

THE DOPPLER EFFECT A MATTER - RADIATION

INTERACTION, CONSEQUENCE OF CONSERVATION LAWS

Stefano Quattrini 11/08/2024

ABSTRACT

The Relativistic Doppler effect was originally derived using Lorentz Transformations. In the same paper the Doppler RADAR or reflecting mirror formula, was proposed, which results in a variation of energy of the radiation in the inertial frame of the RADAR. If also the reflector is kept inertial, the Doppler RADAR formula infringes upon conservation laws, unless a compensating force makes the work to keep the mirror inertial. To avoid resorting to external agents, but complying with conservation laws, Doppler effect must be based on energy and momentum conservation, accounting for recoil which is always present, considering that at least one frame is not inertial.

INTRODUCTION

The Relativistic Doppler effect is mostly considered a transformation law for frequencies between different frames of reference [] found in Einstein's famous paper in 1905, as an application of Lorentz Transformations, derived as well in such script. Consider two inertial frames, IRF0 and IRF1, approaching at a relative speed w, with an ideal mirror attached to IRF1 while an emitter of electromagnetic (EM) waves in IRF0. When EM waves are emitted, they bounce back and the longitudinal Doppler effect, an experimentally verified phenomenon, for approaching bodies, shows that the frequency of the radiation increases after each detection. In the same paper Einstein found the Doppler RADAR formula. Below is the simple one-dimensional case [1],[2],[4] experimentally verified to a certain order of accuracy,

$$fr= f_0(1+\beta)/(1-\beta) \tag{1}$$

were f_o is the frequency of emission in one frame and fr is the reception frequency in the same frame and β=v/c.

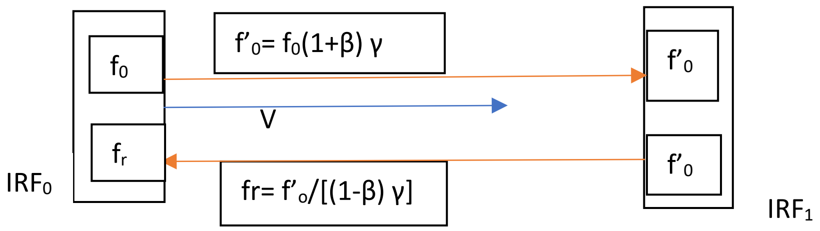

Fig. 1. The Doppler Radar found with Relativistic Doppler formulas

Well known Doppler RADAR applications, use Eq.(1) where the speed of a moving object v=cβ is the quantity to be estimated. The energy content of the initially emitted radiation is E_{ph} = nhf_0 hence From Eq.(1) it is

$$E_r \approx nhf_0(1+\beta)/(1-\beta) =E_{ph}(1+\beta)/(1-\beta) \tag{2}$$

since the RADAR is an inertial frame it is possible to compare the energy emitted with the one received back same as found in [1] (Eq.13) : $\Delta E \equiv Er-E_{ph} = -E_{ph}\ 2\ \beta/(1-\beta)$ (3)

That is basically the work W done on the mirror by radiation in case of inertial motion. The sign is the opposite than the one derived in [1] since here it is considered the final energy minus the initial.

In the case of the object approaching to the RADAR, $\beta < 0$, as the frequency of the radiation increases, in each inertial frame, the energy does the same in the RADAR's frame. An excess energy is detected in RADAR inertial frame, [1] such that: $W=\Delta E > 0$ from Eq.(3).

At low speeds where β is much smaller than one, and E_0 is small, it is reasonable to consider $\Delta E \approx 0$, typically sufficient for practical purposes. The authors in [1] show that the variation of the momentum of the mirror multiplied by c, compensates the work done by the EM waves on the mirror. Although possible to keep the system inertial that way, there is no such compensating momentum in any real case, it can be only a theoretical situation.

APPLICATION OF CONSERVATION LAWS

In general, the energy difference expressed in Eq.(3), for $\beta < 0$ comparable to unity (high speeds), is not negligible and would represent additional energy absorbed by an object stationary in IRF0. As a generic case with an EM wave train, where E_{EM} and P_{EM} are its electromagnetic energy and momentum $\Delta P/P_{EM} = \Delta E/E_{EM} = -2\beta/(1-\beta)$ and considering the same for the RADAR and mirror

$\Delta P/P_{RADAR} = \Delta E/E_{RADAR}$ $\Delta P/P_{mirror} = \Delta E/E_{mirror} = 0$

Only with the help of an external counterbalancing force responsible of an equal and opposite effect of the radiation such as $\Delta E/E = 2\beta/(1-\beta)$, it is possible to keep the system inertial and consider that formula as being exact. Otherwise, to keep a constant speed in the presence of interactions as presented in [1], it is necessary an unrealistic requirement of infinite masses, making the formula not acceptable. The authors [1] forced a compliance with conservation laws but as a matter of fact they accepted that some external energy must be provided to the approaching mirror.

A realistic scenario involves at most one inertial frame where the mirror can make a finite work against the radiation with a recoil to the pulse. Radiation recoil (supported by experimental evidence) is a real effect consequence of momentum-energy conservation of the absorbed radiation. That draws its energy from the kinetic energy of the non-inertial mirror, a finite mass object which slows down, while providing energy to increase the energy content of the EM wave. Considering the RADAR fixed on an embankment (IRF0) and the mirror on a wagon with negligible mass m compared to the embankment (attached to Earth), the wagon must be affected by radiation recoil for how negligible it might be.

With β_0 and β_1 the initial and final relative speeds, $E_0=mc^2$, the rest energy of the mirror, following what stated in [4] from equation (14 and 15), the key equation relevant to energy conservation becomes $E_{ini} = \gamma_0 E_0 + hf_0$, $E_{final} = \gamma_1 E_0 + hf_1$; $h(f_1 - f_0) = E_0(\gamma_0 - \gamma_1)$ with $\gamma_1 = 1/\mathrm{sqrt}(1-\beta_1^2)$,

for momenta it is $P_{ini} = \gamma_0 E_0\, \beta_0/c - hf_0/c$ $P_{final} = \gamma_1 E_0\, \beta_1/c + hf_1/c$; $P_{final} = P_{ini}$

$\gamma_1 E_0\, \beta_1/c + hf_1/c = \gamma_0 E_0\, \beta_0/c - hf_0/c$ or $h(f_1+f_0) = E_0(\gamma_0\, \beta_0 - \gamma_1\, \beta_1)$

the target is to express f_1 in term of γ_0, β_0, E_0, f_0

A significative partial result found also in [4] is $f_1/f_0 = \gamma_0(1+\beta_0)\, \gamma_1(1+\beta_1)$ tells us that the ratio of the frequencies depends on two Doppler effects at the two different speeds.

With some algebraic manipulation (SEE APPENDIX), the ratio of the frequencies becomes

$f_1/f_0 = (1+\beta)/[(1-\beta)(1+2hf/\gamma E_0)] = (1+\beta)/[1-\beta+2hf_0/(\gamma E_0)-2\beta hf_0/\gamma E_0] \approx (1+\beta)/(1-\beta +2\, hf_0/\gamma E_0)$ (4)

that considers the fact that the term $2\beta hf_0/\gamma E_0$ is usually much smaller than $2\, h\, f_0/\gamma_0 E_0$

1) With $\beta = 0$, negligible relative speed → Compton Effect $f_1/f_0 = 1/[1+2E_{ph}/(mc^2)]$

2) Negligible mass of the absorbed radiation $E_{ph} / \gamma_0 mc^2 <<1$, → Doppler Radar Eq.(1)

the energy shift ratio is $\Delta E/E_0 = \Delta f/f_0= 2\, (\beta - E_{ph}/(\gamma_0 E_0))/[1- \beta +2E_{ph}/(\gamma_0 E_0)]$ (5)

The speed cannot remain constant $c\beta$ after every bounce of radiation in case of perfect reflectors. Since the relative variation of the kinetic energy of the mass and the one of radiation must be the same $(\gamma_0-\gamma_1)E_0 = nh(f_1-f_0)$ where $E_0=mc^2$. In this case, the same energy is present, but it is taken from the wagon of mass m_0. $\Delta E_{ph} = E_0\, (\gamma_0-\gamma_1)$, hence $\Delta E/E_0 = (\gamma_0-\gamma_1)$. A net kinetic energy transfer of $(\gamma_0-\gamma_1)E_0 >0$ occurs from the mirror to the photon in the approaching case, on the contrary $(\gamma_0-\gamma_1)E_0 < 0$ there is a net energy transfer from the photon to the mirror in the departing case $nh(f_1-f_0) < 0$. The frequency of the photon decreases, when the mirror departs from the source (RADAR).

The **recoil energy** ΔE_{ph} must always be finite and different from zero, to allow the conservation of energy, but with large masses the **recoil effect** which involve the speed change of at least one mass is usually negligible.

The following equation $(\gamma_0-\gamma_1)E_0 = nh(f_1-f_0)$ is central for the explanation of the effect. If both Radar and mirror are inertial, by definition, $\gamma_0 = \gamma_1 > 1$. No relative speed change is involved, $(\gamma_0-\gamma_1)E_0 = 0$ then $nh(f_1-f_0) = 0$, such that $\Delta f = 0$, no frequency shift with $\beta>0$, no recoil energy.

But since $nh(f_1-f_0) >0$ (non-zero recoil energy), the effect is well approximated experimentally as $\Delta f/f_0= 2\, \beta/(1- \beta) >0$ with $\beta >0$, the $\gamma_0 = \gamma_1$ must be false, hence there is no room for two inertial frames, but just one.

The wagon slows down, diminishing its kinetic energy by ΔE in the centre of mass (COM) of the system (RADAR in this case), thus reducing its relative speed v in the COM by $2E_0/\gamma mc$. The effect of radiation recoil on the embankment is orders of magnitude smaller, virtually negligible, making it acceptable for the embankment to be constantly attached to IRF0. This scenario avoids the perpetuum mobile of the first kind, providing also a sound physical framework for the Doppler effect.

The radiation recoil is strictly dependent on an exchange of energy with a body of a certain mass which loses some kinetic energy in the COM of the system. The Doppler effect which is an effect of matter radiation interaction, must rely only on energy and momentum conservation as a physical foundation. The solution found can be approximated by the formula used for practical purposes.

Using Lorentz Transformations instead, to find the Doppler or Doppler RADAR , relying on the principle of inertial frames, the compliance with the non-negotiable principle of physics is missing. The energy of an isolated system cannot increase over time, even for a limited but finite amount of time, allowing to extract a net energy amount form nothing. Lorentz Transformations used to find the Doppler RADAR formula, where radiation exchange is present, are not properly used. The equivalence of inertial frames, upon which they are based, is unsuitable for such application.

The Doppler Radar formula derived from Lorentz Transformations infringes upon conservation laws and can only serve as a good approximation in real problems, where radiation recoil effects can be neglected (although always present). The Eq.(3) is valid when the energy of radiation is negligible in comparison to the rest energy of the massive reflector: low momentum of radiation.

When (4) reduces to (3) at higher speeds due to the relativistic mass of the mirror, would make the infringement of conservation laws even worse: $\Delta E = 2Eph\ \beta/(1-\beta)$, unless the variation of energy of the radiation corresponds to actual variation of $(\gamma_0-\gamma_1)E0$, where $\gamma_0-\gamma_1$ gets negligibly small, but never 0.

CONCLUSIONS

Applying Lorentz transformations, to a problem involving an exchange of EM radiation between inertial frames, leads to the Doppler Radar formula. It is then configured as a violation of energy conservation laws in each inertial frame. It is impossible to relate two inertial frames with light signals and obtain reliable information at any level of accuracy using Lorentz Transformations, thus the equivalence of inertial frames is falsified by a noncompliance with energy conservation laws. Doppler effect and every problem involving radiation frequency shift, should be treated with at most one inertial frame. Considering a very superficial analysis, Doppler effect might look like an observer dependent phenomenon, but its origin is the exchange of energy momentum between mass and radiation alone. Radiation recoil and variation of momentum and kinetic energy of the reflectors are necessary to give account to any realistic scenario.

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APPENDIX

ENERGY and MOMENTUM PRIOR AND AFTER THE ABSORPTION AT THE RADAR

a) $h(f_1+f_0)= E_0 (\beta_0\gamma_0 -\beta_1\gamma_1)$ b) $h(f_1- f_0)= E_0 (\gamma_0 - \gamma_1)$

c) $\gamma_0= 1/\mathrm{sqrt}(1- \beta_0^2)$, d) $\gamma_1= 1/\mathrm{sqrt}(1- \beta_1^2)$

From (A) $\mathbf{(f_1- f_0) = E_0/h (\gamma_0 - \gamma_1)}$ (a1)

From (B) $(f_1+f_0)=$ $\mathbf{E_0/h (\beta_0\gamma_0 -\beta_1\gamma_1)}$ (b1)

With (a1) and (b1) it is $S=(f_1+f_0)= E_0/h (\beta_0\gamma_0 -\beta_1\gamma_1)$, $D= (f_1- f_0) = E_0/h (\gamma_0 - \gamma_1)$

$f1 /f_0 = (S+D)/(S-D) = [(\beta_0\gamma_0 -\beta_1\gamma_1)+ (\gamma_0 - \gamma_1)] /[(\beta_0\gamma_0 -\beta_1\gamma_1)-(\gamma_0 - \gamma_1)] =$

$[\beta_0\gamma_0 + \gamma_0- \beta_1\gamma_1 - \gamma_1] /[\beta_0\gamma_0 -\beta_1\gamma_1- \gamma_0 + \gamma_1] = [\gamma_0(1+\beta_0) - \gamma_1 (1+\beta_1)] /[(\beta_0\gamma_0 - \gamma_0 + \gamma_1-\beta_1\gamma_1)]$

$f1 /f_0 = \mathbf{[\gamma_0(1+\beta_0) - \gamma_1 (1+\beta_1)] /[\gamma_0 (1-\beta_0)+ \gamma_1(1-\beta_1)]}$ **(e)**

working in **(e)** with equations c and d and considering that $\mathbf{\gamma(1+\beta) =\sqrt{[(1+\beta)/(1- \beta)]}}$

$f1 /f_0 = \mathbf{[\sqrt{[(1+\beta_0)/(1- \beta_0)]} - \sqrt{[(1+\beta_1)/(1- \beta_1)}]/ [\sqrt{[(1-\beta_0)/(1+ \beta_0)]} + \sqrt{[(1-\beta_1)/(1+ \beta_1)]}}$

Numerator $[\sqrt{[(1+\beta_0)(1- \beta_1)]} - \sqrt{[(1+\beta_1)(1- \beta_0)]} / [\sqrt{(1- \beta_0) (1- \beta_1)}] = N/[\sqrt{(1- \beta_0)}\sqrt{(1- \beta_1)}]$

denominator is $N /[\sqrt{(1+ \beta_0)}\sqrt{(1+ \beta_1)}]$.

$f1 /f_0 = \sqrt{[(1+\beta_0)(1+\beta_1)]}/ \sqrt{[(1-\beta_0)(1-\beta_1)]} = \mathbf{\gamma_0(1+\beta_0) \gamma_1 (1+\beta_1)}$ **double doppler** **(f)**

by considering $S+D = (f_1+f_0)+ (f_1- f_0) = E_0/h (\beta_0\gamma_0 -\beta_1\gamma_1) + E_0/h (\gamma_0 - \gamma_1)$

$2f_1/ E_0/h = (\beta_0\gamma_0 -\beta_1\gamma_1) + (\gamma_0 - \gamma_1)$; $2 h f_1/ E_0 = \gamma_0 + \beta_0\gamma_0 -\beta_1\gamma_1 - \gamma_1$;

$\beta_1\gamma_1 + \gamma_1 = \gamma_0 + \beta_0\gamma_0 -2 h f_1/ E_0$; $\gamma_1(1+\beta_1) =$ $\mathbf{\gamma_0 (1+ \beta_0) -2 h f_1/ E_0}$ (g)

$(\gamma_0 (1+ \beta_0) - \gamma_1(1+\beta_1)) E_0/ 2 h = f_1$ the difference of the two doppler effects gives that

replacing (g) in (f) $f_1 /f_0 = \gamma_0(1+\beta_0)$ $\mathbf{[}\gamma_0 (1+ \beta_0) -2 h f_1/ E_0] = (1+ \beta_0)/(1- \beta_0) -2 h \gamma_0(1+\beta_0) f_1/ E_0$

$= (1+ \beta_0)/(1- \beta_0) -2 h \gamma_0(1+\beta_0)(1- \beta_0) f_1/E_0 = (1+ \beta_0)/(1- \beta_0) -2 h f1/\gamma_0E_0$

$f1 /f_0 = (1+ \beta_0)/(1- \beta_0) -2 h f1/ \gamma_0E_0$; $\mathbf{f_1 = f_0 (1+ \beta_0)/(1- \beta_0) -f_0 2 h f1/ \gamma_0E_0}$ **(h)**

solving (g) in f1, $f_1 = [f_0 (1+ \beta_0)/(1- \beta_0)] /(1+2 h f_0/\gamma_0E_0) = f_0 (1+ \beta_0)/[(1- \beta_0)(1+2 h f_0/\gamma_0E_0)]$;

$f_1/ f_0 = (1+ \beta_0)/[(1- \beta_0) (1+2 h f_0/\gamma_0E_0)] = (1+ \beta_0) /(1- \beta_0 +2 h f_0/\gamma_0E_0 - \beta_0 (2 h f_0/\gamma_0E_0))$

$(1+ \beta_0) / (1- \beta_0 +2 h f_0/\gamma_0E_0 - \beta_0 (2 h f_0/\gamma_0E_0)) \approx (1+ \beta_0) / (1- \beta_0 +2 h f_0/\gamma_0E_0)$