

# Why the predictions of Bohmian mechanics agree with those of standard quantum mechanics

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In the form that is currently used most of the time, Bohmian mechanics gives precisely the same results for the the statistics of any conceivable experiment as nonrelativistic standard quantum mechanics.<sup>1</sup> This fact is not appreciated by everybody. Some misconceptions regarding predictions for correlations that arise in systems of identical particles seem to still be “in the air”. Bohmian particles seem to be distinguishable, a notion suggesting to expect, at least on first thought, some differences in predictions.<sup>2</sup>

Therefore, it may be useful to explain the reasons in more detail than just by casual referral to a continuity equation why Bohmian mechanics does not, indeed, produce different predictions from standard quantum mechanics. More precisely, a proof should be provided to switch from beliefs (or the absence thereof) to certainty. I will do so in the following, hopefully without getting too technical. Rather simple arguments (with minor mathematical support) suffice to clarify the issue, and any objection would have to show these arguments to be wrong in order to become credible.

As a result, it will turn out that if we regard Bohmian mechanics as a theory different from standard quantum mechanics (which may be justifiable on grounds to be discussed), this theory makes the same predictions at the empirical level as standard quantum mechanics. Any falsification of Bohmian theory would therefore falsify standard quantum mechanics and vice versa. Preference of one theory over the other cannot then be justified on the basis of experiments. Instead, Occam’s razor might be invoked. The situation is similar to the relationship between Lorentzian ether theory and special relativity, which also are empirically equivalent. But special relativity gets by with one fewer entity, as it does not require the existence of an ether (or a preferred reference system), therefore it would be preferable. Standard quantum mechanics does not require the existence of particle positions at all times – those are the hidden variables of Bohmian mechanics – so it has fewer entities than Bohmian mechanics and would be preferable on these grounds.<sup>3</sup>

## How to obtain predictions for experiments in standard quantum mechanics

The basic equation of standard quantum mechanics, by which I will always mean nonrelativistic quantum mechanics, is the Schrödinger equation. In the position representation, it is a partial differential equation that is first order in time. To set it up, we need the Hamiltonian of the quantum system; to solve it, we need initial and boundary conditions.

The generic way to produce predictions for an experiment is to first specify the initial state via some experimental preparation procedure. Preferably this will be a pure state, i.e., a wave function, rather than a probability distribution of wave functions, describable as a density operator of a mixed state.

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<sup>1</sup>A purist might therefore consider Bohmian mechanics not a different theory but just an alternative interpretation of quantum mechanics, not distinguishable from it by any experiment.

<sup>2</sup>However, even classical particles may be indistinguishable in a statistical sense [1,2] – and still obey Maxwell-Boltzmann statistics... Bohmian particles are indistinguishable in the same statistical sense and their statistics will be determined by the symmetry properties of their wave function.

<sup>3</sup>But that is a criterion for the distinction of *theories*, it need not be considered convincing enough to establish a preference for one *interpretation* of a theory over another...

Boundary conditions and Hamiltonian are specified by the experimental apparatus and the system as well as the interaction between both.<sup>4</sup>

Second, to generate the statistics of measurements of some quantity  $A$  made at time  $t$  after the initial time  $t_0$ , we solve the Schrödinger equation up to time  $t$ , determine the eigenvalues and normalized eigenfunctions of the Hermitean operator  $\hat{A}$  describing the observable to be measured, and express the solution of the Schrödinger equation at time  $t$  as a linear combination of these eigenfunctions.<sup>5</sup> The coefficient  $c(a)$  in that expansion of the eigenfunction corresponding to an eigenvalue  $a$  of the operator  $\hat{A}$  gives the probability amplitude for a measurement of eigenvalue  $a$ .<sup>6</sup> Hence, the only possible results of such an idealized measurement are eigenvalues of the operator associated with the observable, and the probability of  $a$  occurring is given by  $|c(a)|^2$ , if  $a$  is a discrete eigenvalue. If it belongs to a continuous part of the spectrum of  $\hat{A}$ ,  $|c(a)|^2$  is a probability density instead, with the probability for the measured value lying in the interval  $[a, a + da]$  given by  $|c(a)|^2 da$ .

Note that this is a description of the rules only, how to obtain (statistical) predictions within quantum mechanics. Essentially, these rules can be derived from Born's rule that the probability density for position measurements in an  $N$ -particle system described by the wave function  $\psi(x_1, x_2, \dots, x_{3N}, t)$  is given by  $|\psi(x_1, x_2, \dots, x_{3N}, t)|^2$ . The notion of collapse of the wave function need not appear in these rules as long as we are dealing with a single measurement only. However, if we wish to make a subsequent measurement on the system after having performed the first, another rule of quantum mechanics states that the initial wave function for the second measurement is the eigenfunction obtained in the first measurement. This projection of the original wave function onto an eigenfunction of the observed dynamical variable constitutes an essential step of the collapse and is at the heart of the preparation of *initial states* for any experiment, even the first one.

In some experiments, we may not be interested in determining the probability of an outcome but rather wish to find the spectrum of possible results only.<sup>7</sup> Then we will not need the initial wave function and just have to solve an eigenvalue problem.<sup>8</sup> But any measuring problem can, in principle, be decomposed into a number of steps such as the one described above, and leaving out one does not destroy the genericity of the scheme discussed.

To summarize, the basic theoretical problem in order to predict the statistics of an experimentally measured value within standard quantum mechanics consists of solving the Schrödinger equation starting from an appropriate initial state, corresponding to the experimental preparation, and of an expansion of the calculated wave function in terms of the normalized eigenfunctions of the measured observable. The probability distribution of the measured values, which are eigenvalues of the (operator associated with the) observable is obtained by assigning the absolute squares of the expansion coefficients to the corresponding eigenvalues.<sup>9</sup>

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<sup>4</sup>If everything is constrained to happen inside a box, the wave function should become zero wherever one of its positional arguments is at the box boundaries.

<sup>5</sup>Which is possible as the eigenfunctions of a Hermitean operator form a basis of the Hilbert space.

<sup>6</sup>If the eigenvalue is nondegenerate.

<sup>7</sup>An example would be the atomic spectra determined in the early days of quantum mechanics. The determination of the energy levels of the hydrogen atom alone constituted major progress in quantum mechanics, even without explicit knowledge of their probabilities of occupation.

<sup>8</sup>Which is also useful for the time dependent solution, because the complete solution of the eigenvalue problem posed by the stationary Schrödinger equation immediately enables solution of the initial value problem represented by the time dependent Schrödinger equation. Expanding the initial wave function in terms of the eigenfunctions of the Hamiltonian, we obtain the solution of the time-dependent Schrödinger equation via a replacement of the coefficients  $c_n$  of the expansion with  $c_n e^{-\frac{i}{\hbar} E_n (t-t_0)}$ , where  $E_n$  is the eigenvalue of the Hamiltonian corresponding to the  $n^{\text{th}}$  eigenfunction in the expansion.

<sup>9</sup>If there are degenerate eigenvalues, the eigenfunctions have to be chosen orthogonally (in the non-degenerate case,

Now that we are in a position to describe measurement outcomes within standard quantum mechanics, let us consider what a similar attempt would look like in Bohmian mechanics.

## How to obtain predictions for experiments in Bohmian mechanics

Bohmian mechanics has *two* fundamental equations of motion. One is the Schrödinger equation

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H |\psi(t)\rangle . \quad (1)$$

The wave function here describes a system with a fixed particle number  $N$

$$\langle x_1, x_2, \dots, x_{3N} | \psi(t) \rangle = \psi(x_1, x_2, \dots, x_{3N}, t) \quad (2)$$

and is assumed to have the same degree of reality as, say, a field in classical physics.<sup>10</sup> As we shall see, it governs the motion of particles. The second equation is the one describing particle motion (and may be considered a single first-order ordinary differential equation for a  $3N$  dimensional vector)

$$\frac{dx_k}{dt} = \frac{\hbar}{m_k} \Im \left( \frac{\partial_{x_k} \psi}{\psi} \right) = \frac{i\hbar}{2m_k} \frac{1}{\psi^* \psi} \left[ (\partial_{x_k} \psi^*) \psi - \psi^* (\partial_{x_k} \psi) \right] , \quad k = 1, 2, \dots, 3N \quad (3)$$

where for brevity the arguments of the wave function have been suppressed. (They are the same as in Eq. (2).) A fully specified quantum mechanical state consists of a wave function *and* a set of particle positions. It is assumed that the initial wave function can be prepared experimentally just as in standard quantum mechanics. If the positions of the particles could be equally well initialized in an experiment, the set of equations (1) through (3) would allow one to determine the outcome of any position measurement exactly, not just probabilistically. However, preparation of the initial positions is not controllable experimentally in the same way as that of the wave function. Rather, all that is known in a particular experiment, is the probability distribution of initial positions of particles and therefore, in spite of the deterministic nature of the equations, only probabilistic predictions of particle positions will be possible. But this probabilistic aspect is not different from that of classical mechanics: it is only due to a lack of knowledge that the exact positions to be obtained in measurements are indeterminate. In principle, they are determined by the initial state (which is a view that is distinct from that of standard quantum mechanics).

Originally, Bohm [3,4] contemplated the possibility that the probability distribution of the particle positions described by Eq. (3) could be different from  $\psi^* \psi$ , which then would mean that his proposition constitutes a different theory from standard quantum mechanics indeed. It would make different predictions from the latter simply because it would include scenarios where the distribution of particle positions differs from  $\psi^* \psi$ , at the beginning of an experiment and, hence, later. Had that turned out to be the case experimentally, Bohmian mechanics would be able to describe a branch of reality that standard quantum mechanics could not. However, such a situation has never been observed in a verified experiment, so it is customary to assume, as Bohm also did, in order to make his theory

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orthogonality is automatic) and the probability (density) of measuring one of the degenerate values is given by the sum of the absolute squares of the expansion coefficients multiplying the eigenfunctions belonging to that eigenvalue.

<sup>10</sup>That is, the wave function is ontic. Since this wave function is not a function on three-space (plus time) but on configuration space with  $3N$  dimensions, this seems impossible without configuration space itself being ontic. That would, however, appear to imply, that special relativity is only an effective theory. Spacetime would be an epistemic construct, not being ontic. There would be a preferred frame and a true time. The ontology of special relativity would essentially reduce to that of a Lorentzian ether theory, without a way of determining the preferred frame or detecting the ether.

empirically equivalent to standard quantum mechanics, that the so-called *quantum equilibrium hypothesis* holds. This states that in experimentally prepared (initial) states, the probability distribution for the positions of Bohmian particles is always given by

$$w(x_1, x_2, \dots, x_{3N}) = \psi^*(x_1, x_2, \dots, x_{3N})\psi(x_1, x_2, \dots, x_{3N}), \quad (4)$$

where the time argument has been dropped for convenience. There have been attempts to derive, akin to the Boltzmann  $H$  theorem, that a system starting outside of quantum equilibrium will evolve towards it. Moreover, Valentini [5,6] has shown that, if systems exist out of quantum equilibrium, the violation of Bell-like inequalities will permit superluminal communication. If one does not believe the ensuing violations of causality to be possible, the quantum equilibrium hypothesis is a way to avoid them.

In the following, I will always assume Bohmian mechanics to be the theory, in which the quantum equilibrium hypothesis is included as a *postulate* referring to initial states. As it turns out, that hypothesis is not only a necessary but also a sufficient condition for the predictions of Bohmian mechanics and standard quantum mechanics on the statistics of experimental results to become identical. No other conditions are needed and claims to the contrary [7-11] are incorrect [12,13]. I will discuss the first of Ghose's papers in some more detail later. Before, I would like to outline the proof of the assertion that Bohmian mechanics in the sense discussed does indeed make the same predictions as standard quantum mechanics for the results of any experiment.

The proof consists of three steps. First, I will show that the (joint) probability distribution for position measurements (of all particles) necessarily is the same in Bohmian mechanics as in standard quantum mechanics. Then also marginal distributions referring to just a subset of particles of the system must be the same in both theories. Second, I will demonstrate that this implies the probability distribution for the measurement of *any* observable that classically becomes a phase-space function to be the same in standard quantum mechanics and in Bohmian mechanics. Finally, more exotic variables such as the spin are treated in Bohmian mechanics the same way as in standard quantum mechanics, so they do not open room for different predictions on experimental results either.

To demonstrate the first assertion, we have to show that if the joint probability distribution satisfies Eq. (4) at the initial time of an experiment (which it does by the postulate of quantum equilibrium), then the same relation will hold at *any* time, as long as the evolution is by the Schrödinger equation. Obviously, it will be sufficient to show

$$\begin{aligned} & \psi^*(x_1(t+dt), \dots, x_{3N}(t+dt), t+dt) \psi(x_1(t+dt), \dots, x_{3N}(t+dt), t+dt) dx_1(t+dt) \\ & \quad \dots dx_{3N}(t+dt) \\ & = \psi^*(x_1(t), \dots, x_{3N}(t), t) \psi(x_1(t), \dots, x_{3N}(t), t) dx_1(t) \dots dx_{3N}(t), \end{aligned} \quad (5)$$

because this demonstrates that if  $\psi^*\psi$  is the probability density for a measurement of the  $N$  particles at positions  $x_1, \dots, x_{3N}$  at time  $t$ , then  $\psi^*\psi$  will be the correct probability density at time  $t+dt$  for the particles to be at the later positions  $x_1(t+dt), \dots, x_{3N}(t+dt)$ . Taking the ratio of the two volume elements to the left-hand side, we can rewrite Eq. (5) as follows

$$\begin{aligned} & \psi^* \left( x_1(t) + \frac{dx_1}{dt} dt, \dots, x_{3N}(t) + \frac{dx_{3N}}{dt} dt, t+dt \right) \times \\ & \quad \psi \left( x_1(t) + \frac{dx_1}{dt} dt, \dots, x_{3N}(t) + \frac{dx_{3N}}{dt} dt, t+dt \right) J \\ & = \psi^*(x_1(t), \dots, x_{3N}(t), t) \psi(x_1(t), \dots, x_{3N}(t), t) + O(dt^2), \end{aligned} \quad (6)$$

where

$$J = \frac{\partial (x_1(t+dt), \dots, x_{3N}(t+dt))}{\partial (x_1(t), \dots, x_{3N}(t))} \quad (7)$$

is the Jacobian determinant giving the ratio of the two volume elements. This Jacobian can be calculated using the equations of motion for the particle coordinates (3):

$$\begin{aligned} \frac{\partial x_m(t+dt)}{\partial x_k(t)} &= \frac{\partial}{\partial x_k(t)} \left[ x_m(t) + \frac{dx_m}{dt} dt \right] + O(dt^2) = \delta_{mk} + dt \frac{\partial}{\partial x_k(t)} \frac{dx_m}{dt} + O(dt^2) \\ \Rightarrow J &= 1 + dt \sum_k \frac{\partial}{\partial x_k} \left( \frac{i\hbar}{2m_k} \frac{1}{\psi^* \psi} \left[ (\partial_{x_k} \psi^*) \psi - \psi^* (\partial_{x_k} \psi) \right] \right) + O(dt^2). \end{aligned} \quad (8)$$

Moreover, the product  $\psi^* \psi$  at time  $t+dt$  on the left-hand side of Eq. (6) can be expressed via the product at time  $t$  with the help of the Schrödinger equation (1), from which we obtain a continuity equation<sup>11</sup>

$$\frac{\partial}{\partial t} \psi^*(t) \psi(t) + \sum_k \frac{\partial}{\partial x_k} \frac{\hbar}{2m_k i} \left( \psi^* \frac{\partial}{\partial x_k} \psi - \left( \frac{\partial}{\partial x_k} \psi^* \right) \psi \right) = 0, \quad (9)$$

which may then be inserted into

$$\psi^*(t+dt) \psi(t+dt) = \psi^*(t) \psi(t) + dt \frac{\partial}{\partial t} \psi^*(t) \psi(t) + dt \sum_k \frac{\partial}{\partial x_k} (\psi^*(t) \psi(t)) \frac{dx_k}{dt} + O(dt^2), \quad (10)$$

where we have dropped spatial arguments for conciseness. Now we may write out the left-hand side of Eq. (6)

$$\begin{aligned} &\psi^*(t+dt) \psi(t+dt) J \\ &= \left[ \psi^*(t) \psi(t) + dt \frac{\partial}{\partial t} \psi^*(t) \psi(t) + dt \sum_k \frac{\partial}{\partial x_k} (\psi^*(t) \psi(t)) \frac{dx_k}{dt} \right] \left( 1 + dt \sum_k \frac{\partial}{\partial x_k} \frac{dx_k}{dt} \right) \\ &\quad + O(dt^2) \\ &= \psi^*(t) \psi(t) - dt \sum_k \frac{\partial}{\partial x_k} \frac{\hbar}{2m_k i} \left( \psi^* \frac{\partial}{\partial x_k} \psi - \left( \frac{\partial}{\partial x_k} \psi^* \right) \psi \right) \\ &\quad + dt \sum_k \frac{\partial}{\partial x_k} (\psi^*(t) \psi(t)) \frac{i\hbar}{2m_k} \frac{1}{\psi^* \psi} \left[ \left( \frac{\partial}{\partial x_k} \psi^* \right) \psi - \psi^* \frac{\partial}{\partial x_k} \psi \right] \\ &\quad + dt \psi^*(t) \psi(t) \sum_k \frac{\partial}{\partial x_k} \frac{i\hbar}{2m_k} \frac{1}{\psi^* \psi} \left[ \left( \frac{\partial}{\partial x_k} \psi^* \right) \psi - \psi^* \frac{\partial}{\partial x_k} \psi \right] + O(dt^2), \end{aligned} \quad (11)$$

where the last two terms can be combined as the pair of summands add up to a derivative of a product, so we have under the sum  $\frac{\partial}{\partial x_k} \left\{ \psi^* \psi \frac{i\hbar}{2m_k} \frac{1}{\psi^* \psi} [\dots] \right\}$  and the  $\psi^* \psi$  in the numerator cancels that in the denominator. But then the sum of these two terms is just the negative of the second term

<sup>11</sup>To derive Eq. (9), use the product rule on the time derivative of  $\psi^* \psi$  and express the two arising time derivatives via the Schrödinger equation. The two terms containing the potential add up to zero because of the sign change of the imaginary unit on taking the complex conjugate. The remaining two gradient terms in Eq. (9) give the probability current and the preceding derivative with respect to  $x_k$  makes the sum a divergence of the probability current.

on the right-hand side (the  $i$  is in the denominator there and the sign of the parenthesis expression is opposite), so we end up with

$$\psi^*(t + dt)\psi(t + dt) J = \psi^*(t)\psi(t) + O(dt^2), \quad (12)$$

which means that the product of  $\psi^*(t)\psi(t)$  and the comoving volume element in configuration space does not change to linear order in  $dt$ , so its time derivative is zero, no  $\psi^*(t)\psi(t)$  flows into or out of the comoving volume element. Hence,  $\psi^*(t)\psi(t)$  behaves exactly as a probability density (of non-colliding particles in configuration space). This concludes the proof that if  $\psi^*(t_0)\psi(t_0)$  is the probability density of the initial ensemble of Bohmian particle positions,  $\psi^*(t)\psi(t)$  is the probability density of the same ensemble at time  $t$  under the evolution of  $\psi$  governed by the Schrödinger equation. Usually, this problem is short-circuited by referral to the continuity equation (9). My more detailed considerations demonstrate why the validity of this continuity equation, together with the Bohmian equation of motion for particles, Eq. (3), implies that a quantum equilibrium initial distribution remains quantum equilibrium forever. If that was clear to you anyway, all the better for you...

Bohmian mechanics states that measurement of the particle positions will detect the positions of the actual particle configuration, described by Eq. (3), and that the probability distribution for this measurement will be  $\psi^*(x_1, \dots, x_{3N}, t)\psi(x_1, \dots, x_{3N}, t)$ , which is identical to the distribution for position measurements predicted by standard quantum mechanics. So there is always statistical agreement in position measurements of the two approaches. What about measuring other quantities?

Here, the second part of the proof comes into play. All we have to show – and we can copy this from standard quantum mechanics – is that the probability distribution for any observable described by an operator acting in the same Hilbert space as the position operator can be obtained from the probability distribution for position measurements via a standard procedure.

An important quantity in probability theory is the characteristic function, because it contains the full information on a probability distribution, expressed as an expectation value. For a (one-dimensional) random variable  $X$  with probability density  $w(x)$ , this would be defined as

$$F_X(\alpha) \equiv \int_{-\infty}^{\infty} e^{i\alpha x} w(x) dx = \langle e^{i\alpha X} \rangle. \quad (13)$$

Since the absolute value of the exponential function with imaginary exponent is bounded, this expectation value exists for any probability distribution  $w(x)$ . That it contains the full information about the distribution transpires from the fact that it is not only an expectation value but also the Fourier transform of the probability distribution. Inverting it, we obtain

$$w(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\alpha x} F_X(\alpha) d\alpha = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle e^{i\alpha(X-x)} \rangle d\alpha = \langle \delta(X-x) \rangle, \quad (14)$$

where we have made use of a standard representation of Dirac's delta function in terms of an integral over the exponential function with imaginary argument. Now this provides a neat way of switching from one random variable  $X$  to another,  $Y = f(X)$ . The characteristic function of  $Y$  is simply  $F_Y(\alpha) = \langle e^{i\alpha Y} \rangle$  and transforming back to the probability distribution, we find

$$\tilde{w}(y) = \langle \delta(Y-y) \rangle = \langle \delta(f(X)-y) \rangle, \quad (15)$$

which allows us to evaluate the expectation value as an integral using  $w(x)$ . So this is a way of calculating the probability distribution for functions of a random variable from the probability distribution of the random variable itself. The formulas straightforwardly generalize to multidimensional

random variables and have the advantage that the dimension of the vector  $Y$  need not be equal to that of the vector  $X$ , so the formula is more versatile than that based on a transformation of volume elements in configuration space, as we can directly compute marginal distributions via the choice of a random variable of reduced dimension.

What is even more enjoyable, all of this generalizes nicely to quantum mechanics. We just have to replace the random variables by appropriate Hermitian operators and evaluate expectation values the quantum way. In particular, we have the formula

$$w_A(a) = \langle \delta(\hat{A} - a) \rangle = \langle \psi | \delta(\hat{A} - a) | \psi \rangle \quad (16)$$

that provides the probability (density) for a measurement of the observable  $A$  giving the value  $a$ . We may evaluate the expectation value in whatever representation we like. Choosing the position representation, we have

$$w_A(a) = \int_{-\infty}^{\infty} dx_1 \dots \int_{-\infty}^{\infty} dx_{3N} \psi^*(x_1, \dots, x_{3N}, t) \delta(\hat{A} - a) \psi(x_1, \dots, x_{3N}, t), \quad (17)$$

which obviously allows us to determine the probability distribution of measurable values for any operator acting in the Hilbert space of the wave functions on configuration space. Which means, we have a recipe of how to calculate the statistics of other measurements than that of positions in Bohmian mechanics. Well, at least if we can give concrete meaning to the formal result Eq. (17). But that is not so difficult. Let us simply expand the wave function in terms of eigenfunctions of the operator  $\hat{A}$ ,

$$|\psi\rangle = \sum_n c_n |\phi_n\rangle + \int c(a) |\phi_a\rangle da. \quad (18)$$

Herein, the  $|\phi_n\rangle$  are the eigenfunctions from the discrete part of the spectrum of  $\hat{A}$ , normalized according to  $\langle \phi_m | \phi_n \rangle = \delta_{nm}$  and the  $|\phi_a\rangle$  are the eigenfunctions (in the sense of Dirac vectors) from the continuous part, satisfying  $\langle \phi_a | \phi_{a'} \rangle = \delta(a - a')$  (and of course  $\langle \phi_m | \phi_a \rangle = 0$ ). Either the discrete part of the spectrum or the continuous one can be absent but not both. Then we obtain

$$\begin{aligned} \delta(\hat{A} - a) |\psi\rangle &= \sum_n c_n \delta(a_n - a) |\phi_n\rangle + \int c(a') \delta(a' - a) |\phi_{a'}\rangle da' \\ &= \sum_n c_n \delta(a_n - a) |\phi_n\rangle + c(a) |\phi_a\rangle, \end{aligned} \quad (19)$$

which we may rewrite in the position representation and insert into Eq. (17). Using in addition the expansion analogous to (18) for  $\psi^*$ , we finally get, after having carried out all the position space integrals, using the orthogonality of the  $|\phi_n\rangle$  and  $|\phi_a\rangle$ :

$$w_A(a) = \sum_n |c_n|^2 \delta(a_n - a) + |c(a)|^2. \quad (20)$$

Now, this formula has a simple interpretation. It says that the probability of obtaining, on measuring  $A$ , one of the discrete eigenvalues  $a_n$ , is  $|c_n|^2$  (or, if  $a_n$  is degenerate, the sum of all  $|c_n|^2$  corresponding to  $a_n = a$ ) and that the probability density of obtaining  $a$ , if it belongs to the continuous part of the spectrum, is  $|c(a)|^2$  (or, in the case of degeneracy of  $a$ , the sum of several  $|c(a)|^2$ ). We have here derived this result for Bohmian mechanics. However, we know that this is precisely the rule that is valid also in standard quantum mechanics.

All that remains to be considered, are observables, such as for example spin, whose associated operators do not act on the same Hilbert space as position operators. These quantities do not have a classical analog,<sup>12</sup> so their probability distribution cannot be derived from one in configuration space. Bohmian mechanics takes an interesting point of view on the description of variables of this type.

Spin is an *emerging* property, arising through the wave function, not through the particles, which entails that just as in standard quantum mechanics, particles may acquire the property of having a spin through measurement. If we have, for example, an entangled two-particle wave function of two identical particles with total spin zero, then neither of the particles has a spin pointing in a well-defined direction. Measuring one of the two spins will determine its direction and produce a new two-particle wave function that determines the second spin to point in the opposite direction. So while particles in Bohmian mechanics do have, contrary to the situation in standard quantum mechanics, a definite position and velocity, hence momentum, for which the Heisenberg uncertainty relationship only applies because measurement of both cannot be done simultaneously without mutual disturbance, they do not have – and this is the same situation as in standard quantum mechanics – well-defined spin components in orthogonal spatial directions. In standard quantum mechanics, all observables are equal, so to speak, whereas in Bohmian mechanics, some are privileged. Position is a particle property and the position that arises in the fundamental equations is directly measurable. Velocity and momentum are particle properties, but measured velocities and momenta do not normally agree with the values that could be read off the equations at any instant in time.<sup>13</sup> Spin is not a particle property and may even be created by measurement only.

What are, then, the rules that permit the calculation of the statistics of measured values for non-classical observables in Bohmian mechanics? Well, they are obtained by simple extension of the rule (16). Expansion of the wave function into eigenfunctions of the hermitean operator associated with the observable<sup>14</sup> then produces (20) again, i.e., the absolute squares of the expansion coefficients will produce the probabilities or probability densities of measuring the system in the corresponding eigenstate, which in the non-degenerate case is the same as the probability (density) of measuring the corresponding eigenvalue. But this rule is the same as in standard quantum mechanics.

To summarize, in order to predict the statistics of an experimentally measured value within Bohmian mechanics, it is sufficient to solve the Schrödinger equation starting from an appropriate initial state,<sup>15</sup> corresponding to the experimental preparation, and to expand the calculated wave function in terms of the normalized eigenfunctions of the measured observable. The probability distribution of the measured values, which are eigenvalues of the (operator associated with the) observable is obtained by assigning the absolute squares of the expansion coefficients (or sums thereof, in the case of degeneracies) to the corresponding eigenvalues.

This is the same procedure as in standard quantum mechanics. The initial wave function is the same as in standard quantum mechanics, by the quantum equilibrium postulate. The Schrödinger equation to be solved and the boundary conditions are the same, so the wave function from which the result is calculated is the same in both approaches. Then the result must also be the same. There is no way to obtain different outcomes from Bohmian mechanics for observable quantities than from standard quantum mechanics, provided both theories are applied correctly. This holds as long as quantum

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<sup>12</sup>Which would provide a classical phase space description and the possibility to construct an operator representation that acts on configuration space functions.

<sup>13</sup>I will discuss that later.

<sup>14</sup>These will now contain kets acting on a Hilbert space describing internal degrees of freedom, i.e., one that does not correspond to classical phase space.

<sup>15</sup>A solution of Eq. (3) is not even necessary.



equilibrium is postulated as part of the theory.<sup>16</sup>

## Attempts at criticizing Bohmian mechanics

Notwithstanding the knowledge that a proof of the equivalence of predictions from standard quantum mechanics and Bohmian mechanics exists, some authors have tried to escape its conclusions. References [7] through [11] do not constitute a complete list. Unfortunately, none of these approaches have tried to come up with an error in the derivation (which they may not have followed in detail anyway). Rather, they start from a rough idea why there might be a difference and then discuss an experiment, for which they claim that its Bohmian description will give a result different from that of standard quantum mechanics. Invariably, they either apply Bohmian mechanics incorrectly to the problem or even standard quantum mechanics. Reference [13], for example, demonstrates by explicit numerical simulation of the equations of Bohmian mechanics that the claims made in [7] and [9-11] about differing predictions of the two theories on experiments suggested therein do not hold.

Note that there is a more intelligent way of criticizing Bohmian mechanics. This has been done by Englert, Scully, Süssmann, and Walther (ESSW) [14], who suggested a thought experiment with an incomplete Stern-Gerlach interferometer and the storage of which-way information in a microcavity by a single photon, emitted when an atom traverses the cavity. The arrangement of the interferometer has a symmetry plane, and the simplest Bohmian mechanics treatment as a one-particle system (with the stored photon described as a spin!) states that a particle trajectory cannot traverse this plane. Standard quantum mechanics predicts that an atom that has triggered the which-way detector above the symmetry plane will arrive on the screen below it and vice versa. The standard quantum mechanical interpretation thus suggests that particles cross the symmetry plane whereas the Bohmian trajectories do not do so. Therefore, if we measure the position of a particle twice with a sufficient time interval between them, the two positions will not lie on the same Bohmian trajectory, which hence is a metaphysical rather than a physical concept, or surrealistic rather than realistic.

A counter argument of Bohmianists was that the measurement via a which-way detector, which corresponds to a rather microscopic process (a single photon is deposited in the cavity), did not constitute a measurement. Note that ESSW explicitly state that they do not claim Bohmian mechanics to make predictions that differ from those of standard quantum mechanics [15]. Their argument rather attacks the plausibility of the (unobservable) Bohmian particle trajectories. They also do not deny the mathematical existence of these trajectories [16], they just consider their interpretation as particle trajectories doubtful. While a description of the photon as a second particle rather than a spin will allow the Bohmian particle describing the atom to cross the symmetry plane some of the time, the statistics of these crossings still does not agree with standard quantum mechanics where crossing happens in 100 percent of the cases. Bohmianists must, in those cases, where they conclude that the atom did not cross the plane, assign the triggering of the detector on its other side to the wave function producing that interaction rather than the (atom) Bohmian particle. So they succeed in a description of the experiment but with a pretty counterintuitive path taken by the atom and a non-locally triggered detector.

The way to successfully attack the Bohmian interpretation, then, is not via its predictions on observables which cannot differ from those of standard quantum mechanics but by focusing on what

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<sup>16</sup>Once this is done, the theory may be considered a mere reinterpretation of standard quantum mechanics. Because of the hugely different ontologies behind the two interpretations, some will certainly prefer to still consider them different theories. They say very different things about “*reality*” but do not actually make so different statements about *observable reality*.

it says about the non-observable sector of reality, i.e., the Bohmian particle trajectories. In fact, the trajectories *can* be observed in a sense, i.e., they can be made visible. This has been shown in newer work by the Steinberg group [17]. Considering the way Bohmian trajectories are constructed, it is obvious that they just correspond to the path of volume elements of probability, i.e., in a stationary situation they are the streamlines of the probability current density.<sup>17</sup> These streamlines may be determined via weak measurement techniques.<sup>18</sup> If they are considered *average particle trajectories*, we then have a measurement method for Bohmian trajectories [17]! Nevertheless, it is not clear at all to what extent this gives us information about true particle trajectories. In hydrodynamics, streamlines indicate the local direction of motion of fluid volume elements. They can be used to construct trajectories of volume elements. But of particles of the liquid? No. The particles diffuse in addition to the motion indicated by the streamlines and move arbitrarily far away from them. The average may have nothing to do anymore with the true trajectory, after a sufficient amount of time, as particles enter and leave volume elements of liquid. In a continuum description (via a Langevin equation) the trajectories are nondifferentiable, the streamlines are smooth. There is no proof that in quantum mechanics particle trajectories coincide with the streamlines of the probability current density. Hence, an argument that the measurement of these streamlines makes them less surreal as Bohmian particle trajectories may be considered doubtful.

But let me return to the unsuccessful attempts at differentiating on the grounds of apparently diverging predictions between Bohmian mechanics and standard quantum mechanics. I will focus mostly on Ref. [7] by Ghose. Ghose, who is aware of the claim that Bohmian mechanics and standard quantum mechanics produce the same statistical predictions on all experiments, essentially invokes three arguments against this, two of them a bit global in order to justify why to bother with the problem at all, the third consisting in the analysis of an experiment that is supposed to decide between the two theories.

His first argument is that in standard quantum mechanics identical particles are indistinguishable, which leads to certain statistical particularities, while in Bohmian mechanics, Eq. (3) would allow to follow the individual particles from their initial configuration,<sup>19</sup> thereby rendering them distinguishable. He believes that this must lead to differences in predictions for statistical correlations of indistinguishable particles, which leads him to consider a pair of identical particles in a double-slit experiment. Now the proof that Bohmian mechanics and standard quantum mechanics make the same statistical predictions does not rely on any particular symmetry properties of the wave function. It does not use them, hence is independent of the presence or absence of such properties. Moreover, it *demonstrates* that the statistical predictions of Bohmian theory about observable quantities can be obtained without even using the equation (3).<sup>20</sup> Only the wave function is needed, the only equation that *must* be solved in the end is the Schrödinger equation. What Ghose fails to realize is that the hypothetical distinguishability of quantum particles is irrelevant as long as it cannot be realized by consideration of the wave function only. If the wave function has the symmetry properties of bosonic or fermionic particles, then the predictions of Bohmian mechanics on anything observable are those for indistinguishable particles. As far as the wave function is concerned, these particles *are* indistinguishable, and that is all that matters.

Ghose's second general argument is about *ergodicity*.<sup>21</sup> He states that identical statistical predictions

<sup>17</sup>One trajectory in  $3N$  dimensional space can be chopped into  $N$  trajectories in three-dimensional space.

<sup>18</sup>And they may be calculated within standard quantum mechanics by adopting Eq. (3), yet interpreted as an equation of motion for a hypothetical tracer particle (in  $3N$  dimensions) carried along by the probability current.

<sup>19</sup>If we could know that configuration...

<sup>20</sup>While that equation is certainly needed in the proof, it is not needed anymore for predictions from the theory, once it has been shown that these are calculable the same way as in standard quantum mechanics.

<sup>21</sup>For our purposes, ergodicity may be simply defined to mean equality of the time and ensemble averages for all (or

of both theories can only be expected, when the ensemble average (of Bohmian mechanics) is equal to the time average and thus does not hold for a system, in which the equations of motion (Eq. (3)) do not guarantee ergodicity. He proposes to present such a system. Well, it is known that ergodicity is important in statistical mechanics, even though most systems relax to equilibrium in much shorter times than ergodicity arguments would suggest. However, an experimentally measured equilibrium property usually *is* a time average, whereas it is theoretically calculated from an ensemble average. On the other hand, in Bohmian mechanics the equality of ensemble and time average does not seem to play any role at all! Expectation values of an experimentally measured quantity are essentially always just ensemble averages. The average is performed with the state, not on a time sequence. During an experimental time sequence, the quantum state normally changes with each measurement, so a single wave function is not sufficient to calculate the time average of the series, neither in standard quantum mechanics, nor in Bohmian mechanics.

There are two exceptions to this: if the measured quantity satisfies a conservation law, then after the first measurement all subsequent ones will give the same value, because the observable commutes with the Hamiltonian and the wave function will remain an eigenfunction to the observable after the first measurement. In that case, the time average is a constant and the ensemble average will be the same constant, because the wave function remains an eigenstate. The second exception is the quantum Zeno effect. If measurements of a fixed observable are made sufficiently closely timed, then it does not matter, whether the measured observable is a conserved quantity. After the first measurement, the system is in an eigenstate of the observable and if the next measurement follows fast enough, it will still be close to that eigenstate and the measurement in fact forces it back into it. So the sequence of measurements keeps the system in the eigenstate, the measured eigenvalue is always the same again, and we have both a constant time and ensemble average, given by that same eigenvalue. In both cases, ergodicity is automatic. But in general, measurements refer to a given state, i.e., they are made on several systems initialized in the *same quantum mechanical state*. Reference is then *not* to a given *quantum system*, on which repeated measurements are made without preparing the system in a desired initial state in between measurements. Therefore, ergodicity does not play any role. The predictions discussed here refer to measurements on a state. With repeated measurements on a system, successions of wave functions corresponding to state modifications by each measurement must be considered. Of course, if the predictions of Bohmian mechanics and standard quantum mechanics agree for a single state, they will also agree for a succession of states.

Having discarded Ghose's general arguments, let us have a look at the specifics of his experiment. He considers a pair of identical bosons going through a double slit and detected on a screen later. There are symmetry requirements for the wave function both from geometry and from indistinguishability. The (2D) double slit is symmetrical about the  $y$  axis, so the two-particle wave function must satisfy the condition to remain unchanged if both arguments  $x_1$  and  $x_2$  change sign. And it must be unchanged under an exchange of both particles, which is, in fact, already ensured by the ansatz

$$\psi(x_1, y_1, x_2, y_2, t) = \frac{1}{\sqrt{2}} (\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t) + \psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t)) . \quad (21)$$

The geometrical condition then takes the form  $\psi(x_1, y_1, x_2, y_2, t) = \psi(-x_1, y_1, -x_2, y_2, t)$ . Ghose asks, in a somewhat unjustified manner, for a stronger condition, viz.  $\psi_A(x_1, y_1, t) = \psi_B(-x_1, y_1, t)$  and  $\psi_A(x_2, y_2, t) = \psi_B(-x_2, y_2, t)$ . But since that will also lead to the required symmetry of  $\psi$ , we

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almost all) trajectories/histories of the system.

may accept this as a possible realization. It is then easy to derive, using Eq. (3)

$$\left. \frac{dx_1}{dt} \right|_{x_1, y_1, x_2, y_2} = - \left. \frac{dx_1}{dt} \right|_{-x_1, y_1, -x_2, y_2}, \quad (22a)$$

$$\left. \frac{dx_2}{dt} \right|_{x_1, y_1, x_2, y_2} = - \left. \frac{dx_2}{dt} \right|_{-x_1, y_1, -x_2, y_2}, \quad (22b)$$

from which it is seen that if the two Bohmian particles are both on the  $y$  axis, then both their velocities vanish, so they cannot simultaneously cross the axis. Ghose then tries to set up an experimental situation, in which the wave function depends on  $x_1 - x_2$  only, which would lead to

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0, \quad (23)$$

and where the Bohmian particles start symmetrically about the  $y$  axis, so even

$$x_1(t) + x_2(t) = 0. \quad (24)$$

The particles then must be detected as being symmetrically distributed about the  $y$  axis, whereas the standard quantum mechanical prediction would be that there are asymmetric detection events as well. Unfortunately, all of Ghose's conditions cannot be satisfied simultaneously, at least not starting with the original double-slit wave function. So he ends up comparing two situations, where the effective wave function that is needed to set up the situation described by his Bohmian particles is different from that the standard quantum mechanical description is carried out with. Therefore, either Bohmian mechanics or standard quantum mechanics are not applied correctly here, because for the comparison to be meaningful, the (initial) wave function for both must be taken to be the same.

To obtain the desired velocity relation, Ghose notes that far from the slits, the wave function will, due to translational invariance, take the form

$$\psi = \Phi \left( \frac{1}{2}(\mathbf{x}_1 + \mathbf{x}_2) \right) \phi(\mathbf{x}_1 - \mathbf{x}_2), \quad (25)$$

introducing center-of-mass and relative coordinates. We then have, due to Eq. (3)

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{\hbar}{m} \left[ \Im \left( \frac{\partial_{x_1} \Phi}{\Phi} \right) + \Im \left( \frac{\partial_{x_1} \phi}{\phi} \right) \right], \\ \frac{dx_2}{dt} &= \frac{\hbar}{m} \left[ \Im \left( \frac{\partial_{x_2} \Phi}{\Phi} \right) + \Im \left( \frac{\partial_{x_2} \phi}{\phi} \right) \right] = \frac{\hbar}{m} \left[ \Im \left( \frac{\partial_{x_1} \Phi}{\Phi} \right) - \Im \left( \frac{\partial_{x_1} \phi}{\phi} \right) \right], \\ \frac{dx_1}{dt} + \frac{dx_2}{dt} &= 2 \frac{\hbar}{m} \Im \left( \frac{\partial_{x_1} \Phi}{\Phi} \right), \end{aligned} \quad (26)$$

and for this to be zero  $\Phi$  must not depend on either  $x_1$  or  $x_2$  at all. We now have three restrictions on the wave function, it must satisfy Eq. (21), Eq. (25) and  $\Phi$  must depend on the  $y$  coordinates only. A way to satisfy these conditions (and possibly the only one) is to assume  $\psi_A$  and  $\psi_B$  to be plane waves:  $\psi_A(x, y, t) \propto e^{i(k_1 x + k_2 y)}$ ,  $\psi_B(x, y, t) \propto e^{i(-k_1 x + k_2 y)}$ , which leads to

$$\psi(x_1, y_1, x_2, y_2, t) \propto e^{ik_2(y_1 + y_2)} \cos(k_1(x_1 - x_2)). \quad (27)$$

Herein,  $\Phi$  depends on  $y_1 + y_2$  only and  $\phi$  on  $x_1 - x_2$  only, which meets the requirements. Also  $\phi$  is a cosine, describing an interference pattern on the screen. And indeed we have, for each pair of Bohmian particles, the probability distribution of which is proportional to  $|\phi(x_1 - x_2)|^2$  that (23)

holds. But we cannot require (24) because from the distribution proportional to  $\cos^2(k_1(x_1 - x_2))$  many pairs  $(x_1, x_2)$  can be drawn with a sum different from zero. In fact, the sum can be arbitrarily large. So the condition that this sum be zero, used in Ghose's argument, requires a different wave function. In an attempt to achieve it, we might multiply the starting wave function at the slits by the delta function  $\delta(x_1 + x_2)$ .<sup>22</sup> But then the wave function would have a dependence on  $x_1 + x_2$  for most of the interval between the slits and the screen, meaning that Eq. (23) will not hold throughout that interval and the trajectories of the two particles need not be symmetric about the  $y$  axis. Crossing of the axis is then not forbidden by Bohmian mechanics, and Ghose's prediction fails.

The problem here is that the Bohmian ensemble arising from a standard wave function used to describe the double-slit experiment (for simultaneous sending of two identical bosons into the apparatus) does not lead to the requirements (23) nor (24). These requirements hold only for a sub-ensemble of particle pairs, not for all pairs whose probability distribution follows from the wave function. Now it is not inconceivable that an initial wave function may be prepared that will generate the probability distribution for just the subensemble of Bohmian particle pairs considered by Ghose.<sup>23</sup> Then particles from each pair would indeed impinge at symmetric positions above and below the symmetry axis and Ghose's Bohmian prediction would become correct. But – precisely that wave function would also have to be used to produce the prediction from standard quantum mechanics. And of course, that prediction would now agree with the one from Bohmian mechanics. So Ghose's technical mistake can be seen in effectively having used two different wave functions to generate predictions from standard quantum mechanics and from Bohmian mechanics. That cannot lead to a meaningful comparison.

To summarize, a many-particle wave function with the appropriate symmetry properties contains all the correlations resulting from the indistinguishability of identical particles. So Bohmian mechanics will make the correct statistical predictions for that case as well. It is assumed in standard quantum mechanics that the wave function encodes the maximum amount of information that may be available about a system. Which means that it also includes all many-particle correlations. In calculations of Bohmian mechanics that wave function is available. Therefore, all the information that it encodes is derivable in that approach to quantum mechanics, too.

Another attempt at discovering a failure of Bohmian mechanics that I have seen was based on the velocity distribution that can be calculated from the position distribution and Eq. (3). Now while a Bohmianist might even call that quantity the *real* velocity distribution, he would not predict it to be the *measured* velocity distribution. That distribution could be obtained by first defining a velocity operator  $v = \frac{i}{\hbar}[H, \mathbf{x}] = \mathbf{p}/m$  (where the second equality only holds for a non-relativistic Hamiltonian sporting the standard form of kinetic energy) and then determining the probability distribution for measured velocities from the expansion of the wave function in eigenfunctions of  $v$ . There is no reason for this probability to be equal to the distribution of *real* velocities. This may be rationalized by realizing that a velocity measurement typically involves two successive position measurements (and division by the time interval between the two), and after the first measurement, we have a different wave function, so the velocity obtained from both measurements need not be the velocity of the particle detected by the first position measurement. Position and velocity do not commute. An alternative way to determine the probability distribution of velocities would be to first obtain the distribution for position measurements from the prepared wave function, then calculate, for each position, the probability of obtaining a second position result for a wave function

<sup>22</sup>Since one cannot divide by a delta function, the velocity formula (3) cannot be directly applied to a wave function containing a delta function factor. In practice, the delta function would have to be replaced by a very narrow Gaussian or another narrow function approaching the delta function. If needed, the limit of vanishing width of the function may be applied after evaluating the velocities. Note that standard quantum mechanics also cannot use delta functions directly in certain situations. They do not constitute bona fide wave functions, being not normalizable.

<sup>23</sup>That feat does, however, look difficult to me.

evolving from the position eigenfunction determined in the first measurement. This will give the joint probability distribution of two positions, which can be integrated over position pairs with the side condition of constant velocity to obtain the marginal distribution describing the probability density of velocity measurements. For consistency reasons, this procedure should give,<sup>24</sup> within standard quantum mechanics, the same result as the one using velocity eigenfunctions and only the prepared wave function. However, the operations to be performed in Bohmian mechanics are the same, so both procedures should give the same result within that approach, too. But that result would be different from the distribution of *real* velocities. Real positions can be measured directly in Bohmian mechanics, real velocities cannot.

An atomic version of the Hong-Ou-Mandel experiment was also suggested to lead to a situation, in which Bohmian mechanics would not work. The experiment has two identical bosons impinge simultaneously on some kind of half-transparent effective mirror from both sides so that the two possibilities for the particles to appear on opposite sides of the mirror interfere destructively and, hence, both bosons will appear on one side of the mirror (with equal probability for either side). This is the prediction of standard quantum mechanics. The idea behind the suggestion was that the correlations arising due to indistinguishability of the two bosons are not captured by Bohmian mechanics, which would therefore predict a nonzero probability for the bosons to appear on different sides of the mirror, in conflict with what follows from standard quantum mechanics. I discussed in my science education project on ResearchGate how in this case the agreement of the predictions of Bohmian mechanics and quantum mechanics can be visualized using the two-particle wave function. Since ResearchGate no longer supports projects, I have moved the articles published there (including this one) to my personal website. To find the discussion, navigate back from this page to the science education project (once “Up” from the page “About Bohmian mechanics”) and find the entry on “The Hong-Ou-Mandel experiment and Bohmian mechanics”, dated 25 July 2017. There is an introduction and a link to a PDF file with nice pictures.

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<sup>24</sup>It is of course much more complicated than the standard approach via velocity eigenfunctions.

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